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DEFICIENCY SUPPLIED.

KEY

TO THE VARIOUS

ARITHMETICAL WORKS,

NOW EXTANT

BY C. WATERHOUSE.

PORTLAND:
WILLIAM HYDE.
1841.

Educ T 118.41.896

$$\frac{120}{2400} + \frac{60}{2400} + \frac{32}{2400}$$

$$\frac{1}{20} + \frac{1}{40} + \frac{1}{75}$$

H

3 2044 096 993 860

L. D. Wells

20
40
45
185

TYRO'S SPYGLASS,

ARITHMETICAL LEXICON,

AND

SCHOOL TEACHER'S ASSISTANT,

CONSISTING OF VERY IMPORTANT

ARITHMETICAL PROBLEMS,

AND OF THE SOLUTION OF ALL THE

MOST INTRICATE QUESTIONS

TO BE FOUND IN ALL

WORKS ON ARITHMETIC.

BY CHARLES WATERHOUSE.

DURHAM, ME.
PUBLISHED BY THOMAS WATERHOUSE.
1841.

Educ T 11 8.41.876

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PREFACE.

It is well known that common works on Arithmetic abound with abstruse questions, more perplexing than beneficial, and are the same which deprive thousands of scholars of the attention that their teacher might otherwise bestow upon them to better advantage; and practical experience has taught me, that, if a teacher is well versed in arithmetic, he is often perplexed in a manner that can be discribed by a teacher, only, and such perplexities cause an absence of proper ideas to investigate the nature of an arithmetical question proposed by an honorable Tyro, or the artful and designing; also any person of a good retentive ability is at a loss at times of much importance to call to mind the peculiarities of a question that may be taken from any of the many works on arithmetic:

Therefore, knowing such facts, and well considering the situation a school teacher occupies, and in order to relieve him in an arithmetical point of view, I have not spared time nor thought, and have had the help of collegiate educated persons, for the better part of three years past, in compiling this work for the use of school teachers and those wishing to obtain a superior knowledge of arithmetic.

The author is not an advocate of infallibility, yet, since such particular attention has been paid to render this work concise and free from such mistakes as are commonly to be met with in common works on arithmetic, it is to be hoped, that, (though the tongue of the captious caviller should blazon defects for which others might search in vain,) the eye of the candid critic will not see a thing in it which reason and truth would long hesitate to approve.

It consists of very important arithmetical problems that are attended with rules, and the examples to those rules, are all the intricate questions to be found in the different works on arith-

metic, occupying a place with those rules that are suited to their nature, and their solution given.

This work is an indispensable for those to whom it is dedicated, and a complete key to the different works on arithmetic.

It is divided into nine divisions, viz.

INTRODUCTION,	Page	5
PROBLEMS,	"	10
SIMPLE MISCELLANY,	"	16
PROPORTIONALS,	"	17
PERCENTAGE PROPORTIONALS,	"	35
IMPORTANT PROBLEMS,	"	53
MISCELLANY,	"	72
SUNDRY TABLES,	"	85
ARITHMETIC ODDITIES,	"	93

Each of which is systematically subdivided into classes, and the 68th Prob. in IMPORTANT PROBLEMS is of the utmost worth to Surveyors of land and may be considered that discovery concerning local Attraction, which men of science have thought impossible to discover.

INTRODUCTION.

I. ARITHMETIC is the Science, and also the Art of numbers. It has five principal rules for its operation, viz. NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

Numeration teaches to read and note figures. Figures are divided into periods of six figures each, viz. Unit, Million, Billion, Trillion, Quartrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion, Undecillion, and Duodecillion.

The first part of any period is so many units of it; and the latter part so many thousands of it.

TABULAR EXAMPLE OF THE WORTH AND RIGHT ORDER OF EACH PERIOD.

UNITS,	666,777;
TENS,	888,999;
HUNDREDS,	555,666;
THOUSANDS,	444,777;
Tens of thousands,	333,222;
Hund. of "	888,999;
MILLIONS,	144,333
Tens of millions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
BILLIONS,	
Tens of billions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
TRILLIONS,	
Tens of trillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
QUARTRILLIONS,	
Tens of quartrillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
QUINTILLIONS,	
Tens of quintillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
SEXTILLIONS,	
Tens of sextillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	

SEPTILLIONS,	666,555;
Tens of septillions,	444,333;
Hund. of "	222,777;
Thous. of "	888,999;
Tens of thous. of "	222,333;
Hund. of "	444,555;
OCTILLIONS,	
Tens of octillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
NONILLIONS,	
Tens of nonillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
DECILLIONS,	
Tens of decillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
UNDECILLIONS,	
Tens of undecillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	
DUODECILLIONS,	
Tens of duodecillions,	
Hund. of "	
Thous. of "	
Tens of thous. of "	
Hund. of "	

The remaining rules have two natures, viz. Simple and Compound; and for an explicit explanation of both natures of these rules, see common arithmetical works!

II. TO PROVE ADDITION.

Set the excess of nines in each row of figures to the right of its row; and if the excess of nines in the sum's result, and the column made by setting out the several excesses are alike, the work *is right.*

EXAMPLE.	
9876	3
8765	8
7654	4
5432	5
31727	2

III. A TABLE OF SQUARES FROM 21 TO 29.

21×21=441	24×24=576	27×27=729
22×22=484	25×25=625	28×28=784
23×23=529	26×26=676	29×29=841

IV. A MULTIPLICATION TABLE, FROM 4 AND 13, TO 19 AND 19.

	13	14	15	16	17	18	19
4	52	56	60	64	68	72	76
5	65	70	75	80	85	90	95
6	78	84	90	96	102	108	114
7	91	98	105	112	119	126	133
8	104	112	120	128	136	144	152
9	117	126	135	144	153	162	171
11	143	154	165	176	187	198	209

	13	14	15	16	17	18	19
12	156	168	180	192	204	216	228
13	169	182	195	208	221	234	247
14	182	196	210	224	238	252	266
15	195	210	225	240	255	270	285
16	208	224	240	256	272	288	304
17	221	238	255	272	289	306	323
18	234	252	270	288	306	324	342
19	247	266	285	304	323	342	361

V. RULES FOR MULTIPLYING IN A BRIEF MANNER.

1. When the multiplier consists of any number of nines—To the right of the multiplicand write as many cyphers as there are nines in the multiplier—under this new multiplicand, write the given one, (the units, &c. thereof, under the units, &c. of the new one,) then subtract; and this difference is the product, just the same as if the general method had been used. For an example to illustrate the preceding, multiply 987654 by 999.

Thus, 987654000
987654

The product of 987654 multiplied by 999 is equal to 986666346

2. When the multiplier is 13, 14, 15, 16, 17, 18, or 19,—Place the multiplier at the right of the multiplicand, with the sign of multiplication between them—multiply the multiplicand by the unit figure of the multiplier, removing the product one place to the right of the multiplicand; this product and the multiplicand, make the total product.

EXAMPLE.
75964×13
227892
———
987532

3. When the multiplier is 101, 102, &c. to 109—Multiply by the unit figure of the multiplier—remove the product two places to the right of the multiplicand—add together as before for the product.

4. When the multiplier is 111, 112, 113, 114, 115, 116, 117, 118, or 119—Multiply by the unit *figure only of the multiplier*, and add to each mul*tiplication the two figures* which stand next at the *right hand of that* which is multiplied, and to the *two last figures*, separately, add what you carry.

EXAMPLE.
9417
119
———
1120[illegible]23

5. When the multiplier is 21, 31, 41, &c. to 91—Multiply by the ten's figure, only, of the multiplier, and set the unit figure of the product under the place of tens, add them all together, and their sum is the total product.

EXAMPLE.
73918×21
147836
1552278

6. When the multiplier is 22, 23, &c. to 29—Multiply every figure of the multiplicand by the unit figure of the multiplier, and add to each product twice that figure which stands next at the right hand of the figure you multiplied; and to twice the last figure add what you carry.

EXAMPLE.
7657
29
222053

VI. TO PROVE DIVISION.

Add the remainder and all the products of the several quotient figures multiplied by the divisor, together, according to the order in which they stand in the work, and the sum, when the work is right, will be equal to the dividend.

EXAMPLE.

79)987654321(12501953
79+
197
158+
396
395+
154
79+
753
711+
422
395+
271
237+
34+
987654321 Proof.

VII. TABLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

Grains,
24= 1 Pennyweight,
480= 20= 1 Ounce,
5760=240=12=1 Pound.

NOTE.—175 oz. Troy=192 oz. Avoirdupois.

APOTHECARIES' WEIGHT.

Grains,
20= 1 Scruple,
60= 3= 1 Dram,
480= 24= 8= 1 Ounce.
5760=288=96=12=1 Pound.

CLOTH MEASURE.

Nails,
4=1 Quarter,
16=4=1 Yard,
12=3=1 Ell Flemish,
20=5=1 " English,
24=6=1 " French.

NOTE.—2¼ inches=1 Nail.

REFINER'S WEIGHT.

Blanks,
24= 1 Perrot,
480= 20= 1 Mite,
9600=400=20=1 Grain.

NOTE.—The Caret is $\frac{1}{24}$ part of any weight.

AVOIRDUPOIS WEIGHT.

Drams,
16= 1 Ounce,
256= 16= 1 Pound,
7168= 448= 28= 1 Quarter,
28672= 1792= 112= 4= 1 Hundred,
573440=35840=2240=80=20=1 Ton.

PLANETARY MOTION.

Seconds,
60= 1 Minute,
3600= 60= 1 Degree,
108000= 1800= 30= 1 Sign,
1296000=21600=360=12=Zodiac.

TIME.

Seconds,
60= 1 Minute,
3600= 60= 1 Hour,
86400= 1440= 24= 1 Day,
604800= 10080= 168= 7=1 Week,
2419200= 40320= 672=28=4=1 Month,
31557600=525960=8766=365$\frac{1}{4}$=52 w., 1 d., and 6 h.=1 Julian year.
31558154=525969=8766=365$\frac{1}{4}$, 9 m. 14 s.=1 Periodical year.
31556937=525948=8765=365$\frac{5}{24}$, 48 m. 57 s.=1 Tropical "

There are 12 calendar months, with days to each as follows:

The fourth, eleventh, ninth, and sixth,
Have thirty days to each affixed;
All the rest have thirty-one,
Except the second month alone,
Which hath twenty-eight, in fine
'Till leap-year gives it twenty-nine.

LONG MEASURE.

Bar. Corns,
3= 1 Inch,
36= 12= 1 Foot,
108= 36= 3= 1 Yard,
594= 198= 16$\frac{1}{2}$= 5$\frac{1}{2}$= 1 Pole,
23760= 7960= 660= 220= 40=1 Furlong,
190080=63360=5280=1760=320=8=1 Mile.

NOTE.—7$\frac{92}{100}$ in.=1 Link; and 25 Links=1 Pole.

WINE MEASURE.

Cubic Inches,
28$\frac{7}{8}$= 1 Pint,
57$\frac{3}{4}$= 2= 1 Quart,
231= 8= 4= 1 Gallon,
9702= 336= 168= 42=1 Tierce,
14553= 504= 252= 63=1$\frac{1}{2}$=1 Hogshead,
19404= 672= 336= 84=2 =1$\frac{1}{3}$=1 Puncheon,
29106=1008= 504=126=3 =2 =1$\frac{1}{2}$=1 Pipe,
58212=2016=1008=252=6 =4 =3 =2=1 Tun.

SOLID MEASURE.

Inches,
1728= 1 Foot,
46656=27=1 Yard.

NOTE 1.—40 feet of hewn, or 50 feet of round timber=1 ton.
NOTE 2.—16 solid feet=1 foot of wood.
NOTE 3.—8 feet of wood=1 cord.

BEER MEASURE.

Cubic Inches,
35¼= 1 Pint,
70½= 2= 1 Quart,
282= 8= 4= 1 Gallon,
2538= 72= 36= 9= 1 Firkin,
5076=144= 72= 18= 2=1 Kilderkin.
10152=288=144= 36= 4=2=1 Barrel,
15228=432=216= 54= 6=3=1½=1 Hogshead,
20304=576=288= 72= 8=4=2 =1 Puncheon,
30456=864=432=108=12=6=3=2=1 Butt.

ALE MEASURE.

Cubic Inches,
35¼= 1 Pint,
70½= 2= 1 Quart,
282= 8= 4= 1 Gallon,
2256= 64= 32= 8=1 Firkin,
4512=128= 64=16=2=1 Kilderkin,
9024=256=128=32=4=2=1 Barrel,
13536=384=192=48=6=3=1½=1 Hogshead.

DRY MEASURE.

Cubic Inches,
268⅘= 1 Gallon,
537⅗= 2= 1 Peck,
2150⅖= 8= 4= 1 Bushel,
4300⅘= 16= 8= 2= 1 Strike,
8601⅗= 32= 16= 4= 2= 1 Coom,
17203⅕= 64= 32= 8= 4= 2= 1 Quarter,
86016=320=160=40=20=10= 5=1 Wey,
172032=640=320=80=40=20=10=2=1 Last.

SQUARE MEASURE.

Inches,
144= 1 Foot,
1296= 9= 1 Yard,
39204= 272¼= 30¼= 1 Pole,
1568160= 10890= 121= 40= 1 Rood,
6272640= 43560= 4840= 160= 4= 1 Acre,
10144896 00=27878400=3097600=102400=2560=640=1 Mile.

not play any longer, and it was found that the difference of the squares of what each then had was 336. How many did B lose?

Thus, $\overline{14+14}$)336(12 diff.; 14=half sum, and 12÷2=6=half diff. Then 14+6=20 A retired with. And 14—6=8 B retired with; then, 14—8=6 B lost.

PROB. III.—The difference of two numbers, and the difference of their squares, given to find those numbers.

RULE.—Divide the difference of their squares by the difference of their numbers, and the quotient will be their sum; then proceed by PROB. I.

EXAMPLE.

Said Ai to John, Father gave me $12 more than he gave Charles, and the difference of the squares of our separate parcels is 288. How much did he give each?

Thus, 288÷12=24, the sum; then $\overline{24-12}$÷2=6; then 12+6=$18 Ai's share; and 12—6=$6 Charles had given him.

PROB. IV.—The product of three factors, and two of those factors given, to find the third factor.

RULE.—Divide the given product by the product of the two known factors, and the quotient is the factor required.

EXAMPLES.

1. How many days will 9720 lbs. of hay last 12 horses, allowing each horse to eat 45 lbs. per day?

Thus, 12×45=540)9720(18 days. Ans.

2. What must be the length of a lot of land that is 5 rods wide, in order that the lot shall contain one acre?—First consider 16 long, and 10 wide, make 1 acre, therefore they are two factors, and their product is 160; then 5 wide is the given factor. Then to proceed with an example that is of this nature; Divide the product of the two unknown factors by the known factor. Thus, 160÷5=32 rods in length. Ans.

3. What must be the length of a stick of hewn timber that is 10 inches wide, and 1 ft. 3 in. deep, in order to contain 1 ton?—First consider one ton the product of 3 factors. Then, 40 feet =1 ton=69120 in.; and 10 in.×1 ft. 3 in.=150 in. Then 69120÷150=460$\frac{4}{5}$, and 460$\frac{4}{5}$÷12=38$\frac{2}{5}$ feet in length. Ans.

4. Suppose wood to be piled on a base 15 ft. 6 in. long, and 7 ft. 9 in. wide, what must be the height of the pile to contain 16 cords?—First, 16 cords=2048 solid feet, (i. e. a pile 8 feet wide, 8 ft. high, and 32 ft. long.) And 15 ft. 6 in.×7 ft. 9 in. =120$\frac{1}{8}$ ft. Then 2048÷120$\frac{1}{8}$=17,048907 ft. Ans.

PROB. V.—To find a divisor that will divide two or more numbers without a remainder.

RULE.—Divide the larger number by the smaller, and this divisor by the remainder, and thus continue dividing the last divisor by the last remainder till nothing remains, and the last divisor is the divisor sought, if only two numbers are given. But if more than two are given, first find a divisor for any two of the numbers, then find a divisor to the found divisor and another of the given numbers, and thus proceed till all the given numbers are brought in, and the last divisor used is the divisor sought.

EXAMPLES.

1. Suppose a hall to be 154 ft. long and 55 feet wide, what is the length of the longest pole that will exactly measure both the length and width of said hall?

Thus, 55)154(2
110
44)55(1
44
The last divisor = 11)44(4
44

Therefore a 11 foot pole will do it.

2. A owns 720, B 336, and C 1736 rods of land. They agree to divide it into equal house lots, fixing on the greatest number of rods for a lot, that will allow each owner to lay out all his land. How many rods must there be in a lot, how many lots has each, and how many lots in all?

First,
336)720(2
672
48)336(7
336

Then,
48)1736(36
1728
48÷8=6

Lastly,
8 | 720 | 90=A's.
336 | 42=B's.
1736 | 217=C's.
And 349 lots in all.

Therefore 8 rods in each lot.

PROB. VI.—To find the dividend that will contain two or more numbers, given, without a remainder, when either of the given numbers is used as a divisor.

RULE.—Call the numbers the denominators of so many fractions, and a common denominator found thereto in the manner prescribed for finding a common denominator to vulgar fractions, is the dividend sought.

EXAMPLES.

1. Allowing 63 gallons to fill a hogshead, and 42 a tierce, and 32 a barrel, what is the smallest quantity of molasses that can be first shipped in some number of full hogsheads, then discharged and reshipped in some number of full tierces, and again discharged and reshipped in some number of full barrels?

Thus,
2 | 63—42—32
7 | 63—21—16
3 | 9— 3—16
42 | ×3×1×16=2016 gallons. Ans.

2. A certain flour dealer, who purchased his flour on the opposite side of a river, owned three boats, one of which would carry 8 barrels of flour at a time, another 9, another 16. What is the smallest number of barrels he could purchase that would make some number of full freights for either of the boats?

$$\begin{array}{r|l} 4 & 8—9—16 \\ \hline 2 & 2—9—\ 4 \\ \hline \end{array}$$
$$8\times1\times9\times\ 2=144. \text{ Ans.}$$

PROB. VII.—To find the share of each, when the number of shares are not equal, and increase by a given ratio.

RULE.—Divide the sum by the number of persons, the quotient is a middle share; from the middle share subtract the ratio, the remainder is the next share that is less; from the last found share subtract the ratio until you have found all the shares that are less; then to the middle share add the ratio, the sum is the next share that is larger; to the last found share add the ratio till you have found all the shares that are larger.

Divide $600 among five persons, in such a manner that B may have 2 more than A, and C 2 more that B, &c.

$$\$600\div5=\left\{\begin{array}{ll} 116 & \text{A's share,} \\ 118 & \text{B's "} \\ 120 & \text{C's "} \\ 122 & \text{D's "} \\ 124 & \text{E's "} \end{array}\right\}\text{Ans.}$$

PROB. VIII.—To find the share of each, when the number of shares are equal, and increase by a given ratio.

RULE.—Find a middle share by dividing the sum by the number of shares; from the middle share subtract half the ratio, and the remainder is one share—and add half the ratio to the middle share, the sum is another share; these two shares are the two middle shares; for the shares that are less continue to subtract the ratio, and for those that are larger, add the ratio, till the required shares are found.

EXAMPLE.

Divide $1600 among four persons in such a manner that the 2d may have one more than the 1st, the 3d one more than the 2d, &c. Thus, $1600÷4=400, the middle share.

$$\text{Then}\left\{\begin{array}{lll} \$399,50—100 \text{ cts. ratio} & = & \$398,50, \text{ A's share,} \\ 400,00—\ 50 \text{ " } \tfrac{1}{2} \text{ "} & = & 399,50, \text{ B's "} \\ 400,00+\ 50 \text{ " } \tfrac{1}{2} \text{ "} & = & 400,50, \text{ C's "} \\ 400,50+100 \text{ " "} & = & 401,50, \text{ D's "} \end{array}\right\}\text{Ans.}$$

PROB. IX.—The sum of two numbers and their quotient, given to find those numbers.

RULE.—Add 1 to the quotient and divide the sum of the two numbers by this sum, which will give the less number. Subtract the less number from the sum, and you will have the *greater number.*

EXAMPLES.

1. Divide 100 into two such parts that if the greater be divided by the less the quotient may be just 30.

Thus, $30+1)100(3\frac{7}{31}$, less part; then $100-3\frac{7}{31}=96\frac{24}{31}$, greater.

2. The sum of A's and B's ages is 60, and if you divide A's by B's the quotient will be 3. What is the age of each?

Thus, $3+1=4$, then $60\div4=15$ years, B's age; then $60-15=45$ years, A's age.

PROB. X.—The difference of two numbers and the quotient given to find those numbers.

RULE.—Divide the difference of the two numbers by the quotient, less 1, and you will have the less number. Add the less number to the difference, and this sum is the greater number.

EXAMPLES.

1. A greyhound, in pursuit of a hare, run three times as fast as the hare; and when he overtook her, he had run 30 rods more than she. How many rods did each run?

Thus, $30\div\overline{3-1}=15$ rods the hare run; then $15+30=45$ rods the greyhound run.

2. A and B start at opposite points, to skate to the other's starting point: distance 8 miles. A, by having the advantage (hence B the disadvantage) of a uniform wind, performs his task $2\frac{1}{2}$ times the quickest, and 48 minutes the soonest. Required the time that each is skating, and the force of the wind per minute.

Thus, $48\div\overline{2\frac{1}{2}-1}=32$ minutes, A's time; then $32+48=1$ hour and 20 minutes, B's time. Then, $\overline{80+32}\div2=56$ minutes, each, if it had been calm; and $56-32=24$ minutes A was forwarded; and B was retarded the same. Then, if 56 : 8 : : 24 : $18102\frac{6}{7}$ ft., then $18102\frac{6}{7}\div24=754\frac{2}{7}$ ft per minute.

PROB. XI.—To multiply £, *s.* *d.* and *qr.* by £, *s.* *d.* and *qr.*

RULE.—£× by £ are £; £× by *s.* are *s.*; £× by *d.* are *d.*; £× by *qr.* are *qr.*: *s*× by *s.* are 20ths of a *s.*; *s.*× by *d.* are 20ths of a *d.*; *s.*× by *qr.* are 20ths of a *qr.*: *d.*× by *d.* are 240ths of a *d.*; *d.*× by *qr.* are 240ths of a *qr.*; and farthings multiplied by farthings, are 960ths of a farthing.

EXAMPLES.

1. Multiply £19, 19*s.* 11*d.* 3*qr.* by £19, 19*s.* 11*d.* 3*qr.*

	£.	s.	d.	qr.
Thus,	19—	19—	11—	3
	19—	19—	11—	3

$361-361-209-57=19-19-11-3\times19$ £ *s.* *d.* *qr.* £

$\frac{361}{20}-\frac{361}{20}-\frac{209}{20}-\frac{57}{20}=$ " " " "×19 shillings.

£ *s.* *d.* *qr.*

$$\left.\begin{array}{l} 19£ \times 11d. = 209d. \\ 19s. \times 11d. = \frac{209}{20} \text{ of a } d. \\ 11d. \times 11d. = \frac{121}{240} \text{ of a } d. \\ 3qr. \times 11d. = \frac{33}{240} \text{ of a } qr. \end{array}\right\} \begin{array}{l} £\ \ s.\ \ d.\ \ qr. \\ = 19\text{–}19\text{–}11\text{–}3 \times 11d. \end{array}$$

$$\left.\begin{array}{l} 19£ \times 3qr. = 57qr. \\ 19s. \times 3qr. = \frac{57}{20} \text{ of a } qr. \\ 11d. \times 3qr. = \frac{33}{240} \text{ of a } qr. \\ 3qr. \times 3qr. = \frac{9}{960} \text{ of a } qr. \end{array}\right\} \begin{array}{l} £\ \ s.\ \ d.\ \ qr. \\ = 19\text{–}19\text{–}11\text{–}3 \times 3qr. \end{array}$$

£ s. d. qr.

19—0–11–3 $\frac{941}{960}$ sum of the fractions.

380–18—2–2 sum of the whole numbers.

Ans. 399–19—2–1 $\frac{941}{960}$

3. Multiply £3, 6*s*. 8*d*. by £2, 5*s*. 7*d*.

	£3,	6*s*.	8*d*.
	2	5	7
£3×2£ =	6	0	0
s. 6×2£ =	0	12	0
d. 8×2£ =	0	1	4
£3×5*s*. =	0	15	0
s. 6×5*s*. =	0	1	6
d. 8×5*s*. =	0	0	2
£3×7*d*. =	0	1	9
s. 6×7*d*. =	0	0	$2\frac{1}{10}$
d. 8×7*d*. =	0	0	$0\frac{7}{30}$
Ans.	7	11	$11\frac{1}{3}$

Prob. XII.—To divide a larger denomination by a smaller denomination, when the divisor and dividend used in such a division, have the same integer.

Rule.—Reduce the number to be divided to the same denomination the divisor is of. Divide, and the quotient is the answer in the same denomination that the dividend was of before it was reduced to the denomination that the divisor is of.

EXAMPLES.

1. Divide $1 by 4 cents.—Thus, $1=100 cts.; then 100÷4 =$25. Ans.

2. Divide £1 by 1*s*.—First, £1=20*s*. then 20÷1=£20. Ans.

3. Divide 3 bushels by 3 quarts.—First, 3 bush.=96 quarts. then 96÷3=32 bushels. Ans.

4. Divide 4 acres by 5 rods.—First, 4 acres=640 rods; then 640÷5=128 acres. Ans.

SIMPLE MISCELLANY.

1. Part 1500 acres of land between Saul, Seth, and Silas; and give Seth 72 more than Saul, and Silas 112 more than Seth.—Thus, $72+72+112=256$; then $1500-256=1244$, and $1244\div3=414\frac{2}{3}$ acres, Saul's share. Then, $414\frac{2}{3}+72=486\frac{2}{3}$ acres, Seth's share; and $486\frac{2}{3}+72+112=598\frac{2}{3}$, Silas' share.

2. What is the difference between six dozen dozen, and half a dozen dozen?—Thus, a dozen dozen$=144$; then $144\times6=864$; and $12\times6=72$; then $864-72=792$, diff. Ans.

3. The greater of two numbers is 78×25, and their diff. is 15×9. Required their sum and product.—First, $78\times25=1950$, the greater number; then $1950-\overline{15\times9}=1815$, the less; then $1950+1815=3765$, the sum; and $1950\times1815=3539250$, the product.

4. What number, deducted from the 32d part of 3072, will leave the 96th part of the same?—First, $3072\div32=96$; $3072\div96=32$. Then $96-32=64$, which if taken from the 32d part of 3072, leaves the 96th part.

5. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted; and 30 added to the remainder, the half sum shall make 35. Can you tell the number?—Thus, $35\times2-30+20\times5\times7\div3=700$. Ans.

6. A sheepfold was robbed three nights successively; the first night, half the sheep were stolen, and half a sheep more; the second night, half the remainder were stolen, and half a sheep more; the last night, they took half of what were left, and half a sheep more, by which time they were reduced to 30. How many were there at first?—Thus, $\frac{1}{2}+30=\frac{1}{2}$ of what were in the fold before any were taken the last night, therefore $30,5+30,5=61$ in the fold before any were taken the last night, and half of them stolen, and half a sheep more, therefore 31 taken the 3d night, and 62 the 2d, and 124 the 1st; then $30+31+62+124=247$.

7. Take 11 years, 11 mo. 11 weeks, 11 days, 11 hours, 11 m. 11 sec. from 14 years.

	y.	mo.	d.	h.	m.	s.
	14	00	00	00	00	00
	12	1	4	11	11	11
Ans.	1	11	23	12	48	49
Or,	1	11	3w. 2d.	12	48	49 Ans.

8. How many trees, four feet apart each way, may grow on an acre of ground?—First, 1 acre$=43560$ feet, and 4 feet every *way=16 feet, then* $43560\div16=2722$ trees. Ans.

9. At what time, between twelve and one o'clock, do the hour and minute hands of a clock point in directions exactly opposite?—The minute hand must gain 30 minutes on the hour hand, before they will point in opposite directions, and the minute hand, in moving $1\frac{1}{11}$ minute, gains one minute; consequently, $1\frac{1}{11}\times30=32\frac{8}{11}$ m. past 12. Ans.

10. If $\frac{1}{3}$ of 6 be 3, what will $\frac{1}{4}$ of 40 be?—First, $\frac{1}{3}$ of 6 is 2, and $\frac{1}{4}$ of 40 is 10; therefore, if 2 : 3 : : 10 : 15 Ans.

11. Seven-eighths of a certain number exceeds four-fifths of the same by 6; what is that number?—First, $\overline{40\times7}\div8=35$, and $40-35=5$; then $\overline{40\times4}\div5=32$, and $40-32=8$; and $8-5=3$. Then, if 3 : 6 : : 40 : 80, $\frac{7}{8}$ of which exceeds $\frac{4}{5}$ by 6.

12. What is the gross weight of a hogshead of tobacco, weighing neat 11 cwt. 1 qr.; tare 14 lbs. per cwt.—First, $112-14=98$: 112 : : 11 cwt. 1 qr. : 12 cwt. 3 qr. 12 lbs. Ans.

13. The births, in a certain town were, 475, and the proportion 13 boys to 12 girls; what is the number of each?—Thus, $13+12=25$, then 25 : 475 : : 13 : 247 boys, and $475-247=228$ girls.

14. What sum of money will produce as much interest in $3\frac{1}{4}$ years as \$210, 15 can produce in $5\frac{5}{12}$ years?—First, $3\frac{1}{4}$ yrs. $=39$ months, and $5\frac{5}{12}$ yrs. $=65$ months. Then, 39 : 210,15 : : 65 : \$350,25. Ans.

15. Divide the number 360 into four such parts, which shall be to each other as 3, 4, 5, and 6.—First, $3+4+5+6=18$. Then, 18 : 360 : : 3 : 60 } Ans. And, 18 : 360 : : 5 : 100 } A[illegible]
" : " : : 4 : 80 } " : " : : 6 : 120 }

16. C hired A and B to cut wood; A could cut a cord in 4 hours, and C in 6 hours; in what time could both cut a cord?—First, $\overline{6+4}\div2=5$ of 12 hours; then, 5 : 60 : : 12 : 2 hours, 24 minutes. Ans.

PROPORTIONALS.

MOST OF THE FOLLOWING ILLUSTRATE A SEPARATE PRINCIPLE.

1. A lion of bronze, placed at the brink of a reservoir, can spout water into it, from his mouth and right foot. If he spouts from his mouth only, he will fill the reservoir in 50 minutes; if from his foot only, he will fill it in 40 minutes. It has a discharging faucet which will empty it in 25 minutes. In what time would the reservoir be filled, if permitted to spout from his mouth and foot; and in what time would it be filled if the faucet *is left open*, and he spouts from his mouth [illegible] foot?—First,

$\frac{1}{40}+\frac{1}{56}=\frac{2}{200}$ of it filled in 1 minute, then if 9 : 1 : : 200 : 22 minutes. Ans. 1. Then, what runs out in 1 minute, is equal to $\frac{2}{200}$ of what runs in in 1 minute, therefore in 1 minute $\frac{1}{200}$ of the reservoir is filled, when all are running; consequently, 200 minutes=3 hours 20 minutes. Ans. 2.

2. A water-tub that holds 147 gallons, has a pipe that brings in 14 gallons in 9 minutes, and a tap that discharges 40 gallons in 31 minutes; now supposing these both to be left open by mistake at 2 o'clock, and a servant at 5, finding the water running, shuts the tap, only, and is solicitous to know in what time the tub will be filled after the discovery of the accident. What is the reply?—First, from 2 to 5=180 minutes; then, if 9 : 14 : : 180 : 280 gals. runs in, in 3 hours; then, 31 : 40 : : 180 : $232\frac{8}{31}$ gals. runs out in 3 hours; and 280—$232\frac{8}{31}$=$47\frac{23}{31}$ gals. is in at 5 o'clock; then 147—$47\frac{23}{31}$=$99\frac{8}{31}$ gals. is to run in after the discovery. Then, if 14 : 9 : : $99\frac{8}{31}$: 1 hour, 3 minutes, $48\frac{11}{21}$ seconds. Ans.

3. A certain school-master was hired for one month upon these conditions; that if he had 20 scholars, he was to have 25 dollars; and if he had 30 scholars, he was to have 30 dollars. It so happened that he had 29; pray, what was his wages?—If 10 scholars (less 30) : add 25 cents to each (of 20) : : 9 scholars : $22\frac{1}{2}$ cts.; then 25—$22\frac{1}{2}$=$2\frac{1}{2}$ cts. per scholar more than one dollar each, consequently $29,725. Ans.

4. A, B, and C bought a piece of land, the profits of which amount to £120 per annum. Now the sum of money that each paid, was in such proportion that as often as A paid £5, B paid £7, and as often as B paid £4, C paid £6. How much must each have of the gain per annum?—First, if 4 : 6 : : 7 : £10,5; then 10,5+5+7=£22,5; Then, 22,5 : 5 : : 120 : £$26\frac{32}{45}$, A's. Again, 22,5 : 7 : : 120 : £$37\frac{1}{45}$, B's. And, 22,5 : 10,5 : : 120 : £56 C's part of the gain.

5. A person went to 4 taverns in succession; upon entering each of which, he borrowed as much money as he carried to it; and upon leaving them he paid the landlords one dollar each; which done, he finds himself without money. What sum of money did he carry to the first tavern?—First, 100 cents, last landlord's bill, is composed of 50 cts. that he borrowed and 50 cts. that he spent, for he carried to it and borrowed alike; therefore if 100 : 50 : : 50 : 25 cts spent at the third tavern; and if ,50 : ,25 : : ,25 : $0,125 spent at the second tavern; and if ,25 : ,125 : : ,125 : $0,0625 spent at the first tavern; then ,50+,25+,125+,0625=$0,9375. Ans.

6. If 3 men, or 4 women, do a piece of work in 56 days, how long will one man and one women be in doing the same?—First, 3:4::1:1[illegible] to which add 1=$2\frac{1}{3}$; Then, if 4:56::$2\frac{1}{3}$:96 days. *Then, 4:3::1:0*$\frac{3}{4}$, " " " 1=$1\frac{3}{4}$; Or, if 3: " ::$1\frac{3}{4}$: " "

7. A can mow one acre of grass in $5\frac{2}{3}$ hours, and B can mow $1\frac{3}{4}$ acre in $9\frac{1}{5}$ hours. In what time can A and B jointly, mow $8\frac{1}{4}$ acres?—First, $1\frac{3}{4} : 9\frac{1}{5} :: 1 : 5\frac{1}{5}$ hours, B is in mowing 1 acre; then, $5\frac{1}{5}+5\frac{2}{3}=11$ hours; and 11 hours gives 4 acres when both are mowing; and 11 hours=660 minutes. Then, $4 : 660 :: 8\frac{1}{4} : 1361\frac{1}{4}=22$ hours, 41 m. 15 sec. Ans.

8. If a Cardinal can pray a soul out of purgatory, by himself, in 1 hour, a Bishop in 3 hours, a Priest in 5 hours, and a Friar in 7 hours, in what time can they pray out 3 souls, all praying together?—First, $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}=\frac{176}{105}$. Then, if $176 : 105 :: 3 : 1$ h. 47 m. $23\frac{2}{11}$ sec. Ans.

9. A can do a piece of work in 10 days, B in 13 days; in what time could both together finish a piece of work?—Thus, $\frac{1}{10}+\frac{1}{13}=\frac{23}{130}$, and $23 : 130 :: 1 : 5\frac{15}{23}$ days. Ans.

10. A can do a piece of work in 3 weeks, (6 days to a week,) B can do thrice as much in 8 weeks, and C 5 times as much in 12 weeks. If all work together, in what time can they finish a piece of work?—First, $\frac{1}{3}+\frac{3}{8}+\frac{5}{12}=\frac{27}{24}$. Then $27 : 6 :: 24 : 5$ days 4 hours. Ans.

11. A and B can do a piece of work in 5 days; A can do it alone in 7 days. In what time can B do it by himself.—Thus, $7-5=2$, then $2 : 5 :: 7 : 17\frac{1}{2}$ days. Ans.

12. A, B, and C agree to do a piece of work for £2 10*s*.; A and B perform $\frac{3}{11}$ of the work, A and C $\frac{5}{13}$, and B and C $\frac{4}{14}$ of it; How much does each receive, if paid proportionably?—First, $A+B=\frac{3}{11}=,2727$; and $A+C=\frac{5}{13}=,3846$; and $B+C=\frac{4}{14}=,2857$; then $,2727+,3846+,2857=,943$=twice as much as they all performed; therefore $,943\div2=,4715=A+B+C$; then $,4715-,2857=B+C=,1858=A$; and $,4715-,3846=A+C=,0869=B$; and $,4715-,2727=A+B=,1988=C$.

Then, as $,4715 : 50 :: ,1858 : 19s.\ 8d.\ 1\frac{3145}{4715}\ qr.$ A receives,
Also, as " : " :: $,0869 : 9s.\ 2d.\ 2\frac{1570}{4715}qr.$ B "
Again, as " : " :: $,1988 :$ £1, $1s.\ 0d.\ 3\frac{300}{4715}qr.$ C " } Ans.

13. A, B, and C are employed to do a piece of work for $26, 45 cts.; A and B together are supposed to do $\frac{3}{4}$ of the work; A and C, $\frac{9}{10}$; B and C, $\frac{13}{20}$; and are paid proportionably. What is each man's share of the money?—First, $\frac{3}{4}+\frac{9}{10}+\frac{13}{20}=\frac{46}{20}$; separately, $\frac{15}{20}$, $\frac{18}{20}$, $\frac{13}{20}$; and $\frac{46}{20}$ is twice as much as they all did, because each is mentioned twice; then $\frac{46}{20}\div2=\frac{23}{20}$ is what they all did. Then, $\frac{23}{20}-\frac{13}{20}=\frac{10}{20}$ what A did; $\frac{23}{20}-\frac{18}{20}=\frac{5}{20}$ what B did; $\frac{23}{20}-\frac{15}{20}=\frac{8}{20}$ what C did. The sum of the numerators of the parts which each one did, is 23; hence A's part is $\frac{10}{23}$, B's $\frac{5}{23}$, C's $\frac{8}{23}$; then, $23 : 26,45 :: 10 :$ $11,50 A's share of the $26,50 cts.; and, $23 : 26,45 :: 5 :$ $5,75 B's share; and, $23 : 26,50 :: 8 :$ $9,20 C's share.

14. *A and B* can build a boat in 18 days, and with the assist-

ance of C they can do it in 11 days. In what time can C do it?—First, A and B in one day would do $\frac{1}{18}$ of the work, and A, B, and C would do $\frac{1}{11}$ of the work. Then, A, B, and C$=\frac{1}{11}-\frac{1}{18}$ A+B$=\frac{7}{198}$ what C would do in one day; Then, 7 : 198 : : 1 : $28\frac{2}{7}$ days. Ans,

15. A can reap a piece of wheat in 5, days, B can reap it in 8 days, and A and C together in 4 days. How long would all three be in reaping it?—First, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{4}=\frac{23}{40}$, which is made up of 2 days' work by A, and 1 by B and C each; then $\frac{23}{40}-\frac{1}{5}$ (A's extra labor$=\frac{15}{40}$, what all did. Then, 15 : 40 : : 1 : $2\frac{2}{3}$ days. Ans.

16. A and B can dig a trench in 4 days, B and C in 6 days, A and C in 5 days. Required the time that it would take A, B, and C to do it, and the time required for each to do it.—Now A and B, in one day, would do $\frac{1}{4}$; B and C $\frac{1}{6}$; A and C $\frac{1}{5}$; the sum of these parts is $\frac{37}{60}$, and is what all would do in 2 days, because each is mentioned twice; then $\frac{1}{2}$ of $\frac{37}{60}=\frac{37}{120}$, what all would do in 1 day; then if you subtract, separately, what each two would do in one day, from $\frac{37}{120}$, you will have what each one would do in one day, which is $\frac{17}{120}$ for A, $\frac{13}{120}$ for B, and $\frac{7}{120}$ for C. The sum of the numerators=37.

Then,
37 : 120 : : 1 : $3\frac{9}{37}$ days all are in digging the trench,
17 : " : : 1 : $7\frac{1}{17}$ " A is " " " " } Ans.
13 : " : : 1 : $9\frac{3}{13}$ " B is " " " "
7 : " : : 1 : $17\frac{1}{7}$ " C is " " " "

17. If 15 men can perform a piece of work in 11 days, how many men will accomplish another piece of work four times as large, in a fifth part of the time?—First, as 1 : 15 : : 4 : 60 men; then, as 1 : 60 : : 5 : 300 men. Ans.

18. A, B, and C can complete a stair-case in 12 days. A can complete it alone in 23 days, and B in 37 days. In what time can C do it by himself?

First, days 12 : 1 : : 1 day : $\frac{1}{12}$ of it.
As { 23 : 1 : : 1 " : $\frac{1}{23}$ "
37 : 1 : : 1 " : $\frac{1}{37}$ " }

Then, $\frac{1}{12}-\frac{1}{23}+\frac{1}{37}=\frac{131}{10212}$ of it C completes in one day.
Again, 131 : 1 : : 10212 : $77\frac{125}{131}$ days. Ans.

19. A master-mason is offered $25 for doing a piece of work which he can finish in $12\frac{1}{2}$ days; and his journeyman can do it in $18\frac{3}{4}$ days, his apprentice in $22\frac{1}{2}$ days. If they work together, in what time would they finish it, and how much would each earn?—First, all working together would finish $\frac{14}{75}$ of it in one day. Then 14 : 1 : : 75 : $5\frac{5}{14}$ days, all would be in doing it; and the master-mason does $\frac{6}{13}$ of it, the journeyman $\frac{4}{13}$, and the apprentice $\frac{3}{13}$; and if 13 : 25 : : 6 : $\$11\frac{7}{13}$ the master-mason earns,
" : " : : 4 : $\$7\frac{9}{13}$ " journeyman " } Ans.
" : " : : 3 : $\$5\frac{10}{13}$ " apprentice "

20. Bought a load of wood, measuring 2 cords, in sticks of 12 feet in length, and agreed with B to saw the whole to two-feet lengths, at the usual rate of 50 cts. per cord of four-feet lengths. How much should B be paid for sawing it?—First he cut each stick into 4 ft. lengths, hence twice in two; then each of these once in two again, hence each 12 ft. stick is sawed 5 times in two; then, 2 : 50 : : 5 : $1,25 pays B for sawing 1 cord, then $1,25×2=$2,50. Ans.

21. A person looking on his watch, was asked the time of day; he said the time past noon was equal to $\frac{4}{5}$ of the time to midnight; required the exact time.—Thus, 4+5=9; then, 9 : 4 : : 12 : 5 hours and 20 minutes, (i. e.) 20 minutes past 5 o'clock.

22. A person being asked the time of day, said that it was between 3 and 4 o'clock, and the hour and minute hands were together; required the exact time.—Now since the minute-hand moves 12 spaces to the hour-hand one, the minute-hand moves 11 times the fastest; hence, 11 : 1 : : 3 : 3 h. $16\frac{4}{11}$ min. (i. e.) $16\frac{4}{11}$ minutes past 3 o'clock.

23. A son having asked his father's age, the father thus replied: Your age is 12 years, to which if $\frac{5}{8}$ of both our ages be added, the sum will express my age. What was the father's age?—First add $\frac{5}{8}$ of 12 to 12=$19\frac{1}{2}$, which wants $\frac{3}{8}$ of the father's age to complete the answer; hence $19\frac{1}{2}=\frac{3}{8}$ of the father's age. Then if 3 : $19\frac{1}{2}$: : 8 : 52 years, the father's age. Ans.

24. A's age is double that of B, and B's triple that of C, and the sum of all their ages is 140. What is the age of each?—First, call C's age one part of 140 years, B's 3 parts, A's 6 parts; then 1+3+6=10 parts in the sum of their ages. Then 10 : 140 : : 1 : 14 years, C's age: then 10 : 140 : : 3 : 42 years, B's age; and, 10 : 140 : : 6 : 84 years, A's age.

25. A says to B, two-thirds of my age is equal to three-fourths of yours, and their difference is 10 years. What is the age of each?—First, $\frac{2}{3}$, and $\frac{3}{4}$, (reduced to a common denominator)=$\frac{8}{12}$ and $\frac{9}{12}$, therefore their ages are in the ratio of 8 to 9, and 10 years must be equal to $\frac{1}{9}$ of A's age, and $\frac{1}{8}$ of B's; then $\frac{1}{9}$: 10 : : $\frac{9}{9}$:90 years, A's age. Then, $\frac{1}{8}$: 10 : : $\frac{8}{8}$: 80 years, B's age.

26. A man had four sons, whose ages differed from each other 4 years, and the youngest was half as old as the oldest; required the age of each.—First, call 4 years a part, then the third son is 1 part, the second 2, the third 3 parts the most, hence 3 is the youngest and 6 the oldest, for one is half as old as the other.

	1 : 4 : : 3 : 12	years	the	age	of	the	youngest,
Then,	1 : 4 : : 4 : 16	"	"	"	"	"	third,
	1 : 4 : : 5 : 20	"	"	"	"	"	second,
	1 : 4 : : 6 : 24	"	"	"	"	"	first.

27. B's age is $1\frac{1}{2}$ the age of A, and C's $2\frac{1}{10}$ the age of both; and the sum of their ages is 93; What is the age of each?—First, 8=A's age,
then, 12=B's age, Then, 62 : 93 : : 8 : 12 years, A's age,
and, 42=C's age, and, 62 : 93 : : 12 : 18 " B's "
62=their sum. and $30 \times 2\frac{1}{10} = 63$, C's age.

28. A hare is 50 of its own leaps before a pointer, and takes 4 leaps to the pointer's 3; but 2 of the pointer's leaps are equal to 3 of the hare's. How many leaps must the pointer take to catch the hare?—First, 3 : 2 : : 4 : $2\frac{2}{3}$; then $3 - 2\frac{2}{3} = \frac{1}{3}$ of a leap gained every 2 leaps, consequently $\frac{1}{6}$ of a leap at 1 leap; then if $\frac{1}{6}$ of a leap is gained at 1 leap, 50 leaps are gained in 300 leaps. Ans.

29. A hare starts 12 rods before a hound, but is not perceived by him until she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the hound on view, makes after her at the rate of 16 miles an hour. In what time will the hound catch her, and how far will he run?—First, 1 hour=3600 seconds, and 10 miles=3200 rods; then if she runs in 3600 s.: 3200 : : 45 : 40 rods, to which add 12 rods=52 rods, the hare gets before the hound starts; then 16—10=6=1920 rods gained in 1 hour. Then, 1920 : 3600 : : 52 : $97\frac{1}{2}$ seconds in catching the hare; and 3600 : 5120 (rods in 16 miles,) : : $97\frac{1}{2}$: $138\frac{2}{3}$ rods= 2288 feet, the hound runs.

30. The governor of a certain garrison being desirous to know how much money the passage of the garrison amounted to in a certain time, made choice of a loyal servant, giving him orders to receive of every coachman 4 cts.; of every horseman 2 cts.; and of every footman $\frac{1}{2}$ a cent. Now at the end of the term the servant gave the governor $227,50, and let him know that as often as 5 passed in coaches, 9 passed on horseback; and as often as 6 passed on horseback, 10 passed on foot; required the number of passengers of each kind.—First, if 6 : 10 : : 9 : 15 passed on foot as often as 9 passed on horseback, and 5 passed in coaches; hence their numbers were in proportion to each other as 5, 9, 15; and 5 coachmen paid 5 times 4 cts.=20 cts.; 9 horsemen paid 9 times 2 cts.=18 cts.; and 15 footmen paid 15 times $\frac{1}{2}$ ct.=$7\frac{1}{2}$ cts. Therefore the sums which they paid were in proportion to each other as 20, 18, $7\frac{1}{2}$, the sum of which is $45\frac{1}{2}$, consequently the coachmen paid $\frac{40}{91}$, the horsemen paid $\frac{36}{91}$, add the footmen $\frac{15}{91}$. Then, 91 : 227,50 : : 40 : $100, all the coachmen paid, ÷ 4 cts.=2500 coaches. 91 : 227,50 : : 40 : $90, all the horsemen paid, ÷ 2 cts.=4500 horsemen. And 91 : 227,50 : : 15 : $37,50, all the footmen paid, ÷ $\frac{1}{2}$ ct.=7500 footmen.

31. On a certain day, 20 farmers, 30 merchants, 24 lawyers, and 24 tailors, spent at a dinner $64, which was divided among them in such a manner that 4 farmers paid as much as 5 merchants; 10 merchants paid as much as 16 lawyers; and 8 lawyers as much as 12 tailors; how much money did each class pay?—First, the number of each class which paid the same was in proportion to each other as 4, 5, 8, 12. And as often as 4 farmers paid 1 dollar, 1 farmer paid $\frac{1}{4}$ of a dollar, and 20 farmers paid $\frac{20}{4}$=$5. And 4 farmers paid 1 dollar, and 5 merchants the same, therefore 30 merchants paid $\frac{30}{5}$=$6. And 5 merchants=8 lawyers, hence 24 lawyers=$\frac{24}{8}$=$3. And 8 lawyers=12 tailors, therefore 24 tailors=$\frac{24}{12}$=$2. Hence the sums which they paid were in the proportion to each other as 5, 6, 3, 2, and the sum of these numbers is 16; therefore the farmers paid $\frac{5}{16}$, the merchants $\frac{6}{16}$, the lawyers $\frac{3}{16}$, and the tailors $\frac{2}{16}$.

Then,
16 : 64 : : 5 : $20, what the farmers paid;
16 : 64 : : 6 : $24, " " merchants "
16 : 64 : : 3 : $12, " " lawyers "
16 : 64 : : 2 : $8, " " tailors "
} Ans.

32. If the earth turns upon its axis once in 23 hours, 56m. 4s. and if its circumference is 25000 miles, at what rate per hour are the inhabitants at the equator carried?—Thus, 23h. 56m. 4s.:25000 miles::1h.:1044$\frac{11196}{21541}$ miles. Ans.

33. If the sun move, every day, one degree, and the moon thirteen, and at a certain time the sun be at the beginning of Cancer, and in three days after, the moon at the beginning of Aries, the place of their next following conjunction is required.

There are 30° in a sign, and the signs Taurus and Gemini occur between Aries and Cancer; and in three days after, the moon is at the beginning of Aries; therefore the moon lacked 39° of Aries, hence 129° between the two sphere's motions; and 13—1=12° per day, one sphere gains upon the other, and 129° to be gained. Hence, 12 : 1 : : 129° : 10° 45′ of Cancer.

34. Suppose a meteor to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second; in what time would it move round the earth?—Suppose the earth's diameter 8000 miles, then $\overline{8000+50}\times2$=8100, the diameter of the circle described by the meteor. Then as 113 : 355 : : 8100 : 25446,9 miles nearly, its circumference, and 25446,9÷20=1272,345 seconds=21′ 12″ 19‴. Ans.

35. If the mean diameter of the earth's annual path round the sun is 191263000 miles; required its mean motion per minute?—First, 191263000×3,1416=600871840,8 miles, the orbit's circumference. Then, as 365,25d. : 600871840,8 : : 1′ : 1142,44 miles. Ans.

36. When the days are 13$\frac{3}{8}$ hours long, a traveller performs

his journey in $35\frac{1}{2}$ days; in what time will he perform his journey when the days are but $11\frac{9}{10}$ hours long?—First, $\frac{5}{8}$, $\frac{9}{10}$, and $\frac{1}{4}$, $=\frac{25}{40}$, $\frac{36}{40}$, and $\frac{20}{40}$; then $13\frac{25}{40} : 35\frac{20}{40} :: 11\frac{36}{40} : 40\frac{615}{952}$ days. Ans.

35. A and B travel the same direction for the same place, and travel together at B's uniform speed for 9 days, when A turned and went back at B's rate of speed. Then turns again and pursues B at the rate of 18 miles per day, and overtakes him in $22\frac{1}{2}$ days. What is B's rate of speed per day?—Now since they are 9 days in company at their outset, and A travels back, but B keeps on, it shows that B travels 18 days the longest in a right line; and since A overtakes B in $22\frac{1}{2}$ days, it shows that $18\times22\frac{1}{2}=405$ miles that B travels, and $22\frac{1}{2}+18=40\frac{1}{2}$ days in travelling it. Then, if $40\frac{1}{2} : 405 :: 1 : 10$ miles, B's speed per day. Ans.

36. A and B are on a straight road and on opposite sides of a gate; A is distant from it 308 yards, B 277 yards; travelling each towards the gate. A walks $2\frac{1}{3}$ yards, and B 2 yards per second. How long must they walk to make their distances equal from the gate?—First $2\frac{1}{3}-2=\frac{1}{3}$ of a yard gained by A per second, and $308-277=31$ yards to be gained. Then if $\frac{1}{3} : 1 :: 31 : 1$ m. 33 sec. Ans.

37. A man was to travel 335 miles; at the expiration of 7 days he found that the distance which he had to travel was equal to $\frac{25}{42}$ of the distance which he had already travelled. How many miles per day did he travel?—Thus, $42 : 7 :: 25 : 4\frac{1}{6}$, then $7+4\frac{1}{6}=11\frac{1}{6}$, and $335\div11\frac{1}{6}=30$ miles a day.

38. A pedestrian, for a wager of $1000 having engaged to travel 17 miles in 1 h. $34\frac{1}{2}$ m. finished 10 miles in 1 h. 0m. 30s. and performed the task in 1 h. 31 m. It is required to know whether he is before or after time when he has finished 10 miles, and how much before time when he finished?—First, if 1h. 34m. 30s. : 17 :: 1h. 0m. 30s. : 10 miles and 1555 yards, then 10 m. 1555 yds—10m.=1555 yds. after time at the end of 10 miles; and 1h. $34\frac{1}{2}$m.—1h. 31m.=$3\frac{1}{2}$m. Then 1h. $34\frac{1}{2}$m. : 17 :: $3\frac{1}{2}$m. : 1108 yards before time when he finished.

39. There is an island 50 miles in circumference, and 3 men start together to travel the same way arouud it. A travels 7 miles per day, B 8, and C 9. In what time will they be together again; how far and how many times round will each travel, and in what time will C catch A the 3d time?

First, $50\times7+50\times8+50\times9\div7+8+9=50$ days before all will be together again; then $50\times7=350$ miles A travels, and $50\times8=400$ miles B travels, and $50\times9=450$ miles C travels;

Then, $\left\{\begin{array}{l} 50)350(7 \text{ times round A travels} \\ 50)400(8 \text{ " " B "} \\ 50)450(9 \text{ " " C "} \end{array}\right\}$ before all are together again.

And 9—7=2 miles per day that C gains upon A. Then if 2 : 1 : : 50 : 25,+50=75 days to catch A the third time.

40. Suppose two steamboats to start at the same time from places 300 miles apart on the same river, the one proceeding up stream is retarded by the current 2 miles per hour, the other moving down stream is accelerated the same; in still water their engines propel them 8 miles per hour. Now how far from each starting place will the boats meet?—First, both boats in motion gives 16 miles per hour. Then 16÷2=8, and 8+2=10 the speed of one; then 8—2=6, the other's speed, and 16 : 300 : : 10 : 112½ miles one proceeded, then 16 : 300 : : 6 : 187½ miles the other proceeded.

41. A ship of war sailed with 650 men, and provision for a cruise of 15 months. At the end of 3 months she captured an enemy's vessel, and put 75 men on board of her. Five months after, she captured and sunk another vessel, and took on board the crew, consisting of 350 men. How long did the provision last from the commencement of the cruise?—First, 15—3=12, then 650—75=575; then 650 : 12 : : 575 : $13\frac{13}{23}$ months the provision would last 575, after 75 was put on board the first ship; then 575+350=925, and $13\frac{13}{23}$—5=$8\frac{13}{23}$; then 575 : $8\frac{13}{23}$: : 925 : $5\frac{12}{37}$ months; then 3+5+$5\frac{12}{37}$=$13\frac{12}{37}$ months. Ans.

42. A ship's crew of 300 men were so supplied with provision for 12 months, that each man was allowed 30 ounces per day; but after sailing 6 months, they find it will take 9 months more to finish their voyage, and 50 of the crew have been lost. Required the daily allowance of each man for the last 9 months. First, if 6 : 30 : : 9 : 20 oz., then 300 : 20 : : 250 : : 24 oz. Ans.

43. Supposing the diameter of the propelling wheels of a steamboat to be 15 feet, and to make 390 revolutions in a minute, what distance will the boat move forward in 1 hour if the velocity of the boat is the same as the outer surface of the propelling wheels?—First, as 113 : 355 : : 15 : 47,1238 feet, their circumference, then 47,1238×390×60=2088,441 miles. Ans.

44. A, B, and C have among them 135 guineas. A's+B's are to B's+C's as 5 to 7, and C's—B's is to C's+B's as 1 to 7. How many had each.—Suppose A's+B's=50; then, 5 : 50 : : 7 : 70; then 7 : 70 : : 1 : 10, and 70—10÷2=30 B's,+10=40 C's, and 30—10=20 A's, then, A's 20+B's 30+C's 40=90, and if 90 : 20 : : 135 : 30 A's part, then 90 : 30 : : 135 : 45 B's part, and 90 : 40 : : 135 : 60 C's part.

45. A, B, C and D gained $12; A was to have $\frac{1}{3}$ of it, B $\frac{1}{4}$, C $\frac{1}{6}$, D $\frac{1}{8}$. How much has each?—First, $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{21}{24}$, and $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=\frac{8}{21}, \frac{6}{21}, \frac{4}{21}, \frac{3}{21}$; then, 21 : 12 : : 8 : $4,571$\frac{3}{7}$ belong to A, and, 21 : 12 : : 6 : $3,428$\frac{6}{7}$ belong to B, also, 21 : 12 : : 4 : $2,285$\frac{5}{7}$ belong to C, and, 21 : 12 : : 3 : $1,714$\frac{2}{7}$ belong to D.

46. A and B hired a coach in Portland to go 50 miles, for \$25, with liberty to take in C and D when they pleased, When they had gone 15 miles they admit C, who wished to go the same route, and on their return within 25 miles of P. they admitted D for the remainder of the journey, Now each person is to pay in proportion to the distance he rode, and it is required how much each must pay. First, 50 out+50 back=100m. the distance, then $\frac{15}{100}=\frac{3}{20}$ A and B rode before admitting any one, then A, B, and C rode 60 m.=$\frac{3}{5}$ each, before admitting D, then A, B, C, and D rode 25 m.=$\frac{1}{4}$ each when together; then the quotients \$1,87$\frac{1}{2}$, \$5,00, and \$1,56$\frac{1}{4}$, that arises from \$25 being divided severally by $\frac{3}{20}$, $\frac{3}{5}$, $\frac{1}{4}$, and their quotients again, severally by 2, 3, 4, is added to show what each is to pay as follows: 187$\frac{1}{2}$+500+156$\frac{1}{4}$=\$8,43$\frac{3}{4}$ A's, and B's the same, then D's=\$156$\frac{1}{4}$, to which add \$5,00=\$6,56$\frac{1}{4}$ C's part to pay.

47, C, D. and E company and put in together \$3860; C's money was in 3 months, D's was in 5, and E's 7 months; they gained \$234, which was so divided, that $\frac{1}{2}$ of C's gain was equal to $\frac{1}{3}$ of D's, and $\frac{1}{3}$ of D's was equal to $\frac{1}{4}$ of E's. What did each put in?—First, allow C 2 shares, D 3, and E 4, then 2+3+4=9 shares in all. Then 9 : 234 : : 2 ; \$52 C's share of the gain, and 9 : 234 : : 3 : \$78, D's share, and 9 : 234 : : 4 : \$104 E's share, then C's money was in 3 months, D's 5, and E's 7, then \$52÷3=17$\frac{1}{3}$, and \$78÷5=15$\frac{3}{5}$ and \$104÷7=14$\frac{6}{7}$, then 17$\frac{1}{3}$+15$\frac{3}{5}$+14$\frac{6}{7}$=47$\frac{83}{105}$,

Then, 3860 : 1 : : 47$\frac{83}{105}$: \$1400 C put in, and gained \$52
3 : 52

3860 : 1 : : 47$\frac{83}{105}$: \$1260 D put in, and gained \$78
5 : 78

3860 : 1 : : 47$\frac{83}{105}$: \$1200 E put in, and gained \$104
5 : 104

} Ans.

48. A, B, and C are to share £100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively, but C dying, it is required to divide the whole sum properly between the other two.

Thus, as $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$: £100 : : $\left\{\begin{array}{l}\frac{1}{3} : 42\frac{26}{47} \text{ A's}\\ \frac{1}{4} : 31\frac{43}{47} \text{ B's}\\ \frac{1}{5} : 25\frac{26}{47} \text{ C's}\end{array}\right\}$ Again, $\frac{1}{3}+\frac{1}{4}$: $25\frac{25}{47}$: : $\frac{1}{5}$: $14\frac{582}{947}$.

Then $42\frac{26}{47}+14\frac{582}{947}$=£57 2*s*. 10$\frac{1}{4}$*d*. A's share of the £100, and £100—£57 2*s*. 10$\frac{1}{4}$*d*.=£42 17*s*. 1$\frac{3}{4}$*d*. B's share of the £100.

49. A family of 10 persons took a large house for $\frac{1}{2}$ of a year, for which they agreed to pay \$500 rent. At the end of 14 weeks, they took in 4 new lodgers; and after 3 weeks, 4 more; and so on at the end of every 3 weeks during the term, they *took in 4 more,* How much rent must one of each class pay?—*First find a middle term* for each of the several proportional state-

ments, by dividing the rent, severally, by 26, 22, 18, 14, and 10; then use 26 weeks for the first term in each statement, and 3, 3, 3, 3, and 14, severally, for the third terms. Thus,

$500 \div 26 = 19{,}23\frac{1}{5} \times 3 \div 26 = \$2{,}218\frac{158}{169} =$ 5th class each.

$500 \div 22 = 22{,}72\frac{8}{11} \times 3 \div 26 = \$2{,}622\frac{198}{286} + 2{,}218\frac{158}{169} = \$4{,}841\frac{591}{1859} =$ 4th class each.

$500 \div 18 = 27{,}77\frac{3}{9} \times 3 \div 26 =$ diff. between 1 of 4th and 1 of 3d class, to which add 1 of 4th class $= \$8{,}046\frac{2458}{5577} =$ 1 of 3d class.

$500 \div 14 = 35{,}71\frac{3}{7} \times 3 \div 26 =$ diff. between 1 of 3d and 1 of 2d class, to which add 1 of 3d class $= \$12{,}167\frac{12487}{39039} =$ 1 of 2d class.

$500 \div 10 = 50{,}00 \times 14 \div 26 =$ diff. between 1 of 2d and 1 of 1st class, to which add 1 of 2d class $= \$39{,}090\frac{15490}{39039} =$ 1 of 1st class.

50. A certain sum of money is to be divided between 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is 28 dollars. What is the sum?—First. $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{9}{12}$, therefore \$28 $=\frac{3}{12}$ of the required sum. Then, if 3 : 28 : : 12 : \$112. Ans.

51. A, B, C and D spend 35*s.* at a reckoning, and, being a little dipped, agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$. Now what should each pay?—First, $\frac{2}{3}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{21}{12}$, and severally, $\frac{8}{21}$, $\frac{6}{21}$, $\frac{4}{21}$, $\frac{3}{21}$.

Then,
21 : 35 : : 8 : 13*s.* 4*d.* A should pay,
21 : 35 : : 6 : 10*s.* 0*d.* B " "
21 : 35 : : 4 : 6*s.* 8*d.* C " "
21 : 35 : : 3 : 5*s.* 0*d.* D " "
} Ans.

52. A gentleman bought a chaise, horse, and harness for £100; the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{5}$ more than the horse; What was the price of each?—First let the price of the harness consist of 12 parts, then the price of the horse would consist of 15 parts, and the chaise 20 parts; then 12+15+20=47 parts in all.

Then,
47 : 100 : : 12 : £$25\frac{25}{47}$, price of the harness,
47 : 100 : : 15 : £$31\frac{63}{47}$, " " " horse,
47 : 100 : : 20 : £$42\frac{26}{47}$, " " " chaise.
} Ans.

53. A received $\frac{1}{6}$ of a legacy, B$\frac{1}{10}$, and C the remainder; and when they had counted what they received, A had \$80 more than B. How much did each receive?—First, A's share$=\frac{5}{30}$, B's $\frac{3}{30}$, and C's $\frac{22}{30}$; and $\frac{5}{30}-\frac{3}{30}=\frac{2}{30}=\$80$, and $\frac{30}{30}=\$1200$.

Then,
30 : 1200 : : 5 : \$200, A's share of the legacy,
30 : 1200 : : 3 : \$120, B's " " " "
30 : 1200 : : 22 : \$880, C's " " " "
} Ans.

54, A, B, C, D and E gained \$4050, which was so shared that $\frac{1}{2}$ of A's part is equal to $\frac{1}{4}$ of B's, $\frac{1}{5}$ of C's, $\frac{1}{6}$ of D's, and $\frac{1}{8}$ of E's. How much of the gain belonged to each?—First, consider B to have 4 shares, C 5, D 6, and E 8; consequently A would have 8, and 8+4+5+6+8=31 shares in all.

Then if 31 : 4050 : : 8 : 1045\frac{5}{31}$, belonged to A;
" : " : : 4 : 522\frac{18}{31}$, " " B;
" : " : : 5 : 653\frac{7}{31}$, " " C; Ans.
" : " : : 6 : 783\frac{27}{31}$, " " D;
" : " : : 8 : 1045\frac{5}{31}$, " " E;

55. A owns $\frac{7}{8}$ of an estate, and B the remainder; A sells $\frac{3}{4}$ of his part to C, and C sells $\frac{3}{8}$ of his to D, who also buys $\frac{1}{2}$ of A's remaining part. Required the share of each in the estate, and how much they would severally receive on selling it for $14720?—First, B's share is $\frac{1}{8}$ of the estate, and A retains $\frac{11}{64}$ of his part of the estate, and C retains but $\frac{193}{256}$ of what he receives of A, and D receives $\frac{63}{256}$ of it from C; hence A's share $=\frac{44}{376}$, B's $=\frac{32}{376}$, C's$=\frac{193}{376}$, and D's$=\frac{107}{376}$.

Then, 376 : 14720 : : 44 : 1722\frac{208}{376}$ A receives;
" : " : : 32 : 1252\frac{288}{376}$ B "
" : " : : 193 : 7555\frac{280}{376}$ C " Ans.
" : " : : 107 : 4188\frac{352}{376}$ D "

56. If 18 grains of silver will make a thimble, and 12 dwt. a teaspoon, how many thimbles and teaspoons, of each an equal number, can be made from 15 oz. 6 dwt?—First, one thimble and one teaspoon=306 grains, and 15 oz. 6 dwt.=7344 grains. Then if 306:1::7344:24 of each. Ans.

57. A and B have sheets of paper. A said to B, if I had 2 sheets more, I should have twice and half as much as you. B said to A, if I had 4 sheets more I should have half as much as you. Required the number of sheets that each had.—First, if $\frac{1}{2}$ wants 4, $2\frac{1}{2}$ wants 20 to make it $\frac{1}{2}$; and if $\frac{1}{2}$ is 20, $2\frac{1}{2}$ is 50, A's number of sheets with the number that he wanted to make his number equal twice and half as much as B's number of sheets; then $50 \div 2\frac{1}{2}=20$ sheets that B has; then $\overline{20+4}\times 2=48$ that A has.

58. A, B and C agree to contribute $730 towards the building of a church, at the distance of 2 miles from A, $2\frac{7}{8}$ miles from B, and $3\frac{1}{2}$ miles from C; and they agree, that their shares shall be reciprocally proportional to their distances from the church; required what each contributes.—First, $2\times 8=\frac{16}{8}$, then $\overline{2\times 8+7}=\frac{23}{8}$, and $3\frac{1}{2}=\frac{28}{8}$, therefore the first pays $28 as often as the second $23, and the third $16; then C $\overline{16}$+B $\overline{23}$+A $\overline{28}$=67;

Then, 67 : 730 : : 16 : $174,32$\frac{56}{67}$ C must pay;
" : " : : 23 : $250,59$\frac{47}{67}$ B " "
" : " : : 28 : $305,07$\frac{31}{67}$ A " " Ans.
Proof, $730

59. The sides of two square pieces of ground are as 3 to 5, *and the sum of* their superficial contents is 30600 square feet. *What is the length of* a side of each piece?—First, $3\times 3=9$; *then* $5\times 5=25$; *and* $25+9=34$; and 34 : 30600 : : 9 : $\sqrt[2]{8100}$

$=90$ feet, side of the smaller piece; then $34 : 30600 :: 25 : \sqrt[2]{2250}=150$ feet, side of the greater.

60. G and H buy 48 acres of land at \$12 per acre, of which H is to have a piece containing 12 acres, which G and H think to be $\frac{1}{3}$ better than 12 of the 36 that G is to have. How much should each pay?—Since G's land is $\frac{3}{4}$ of the whole, he receives $\frac{3}{4}$ of the $\frac{1}{3}$ better; and $48\times12=\$576$ the land cost; and $\frac{1}{3}$ of the cost of 1 acre is \$4, and $12\times4=\$48$, $\frac{3}{4}$ of which is paid by H; also $12\times12=\$144+36=\$180$ paid by H; then $48 : 576 :: 36 : 432$, then $432-36=\$396$ paid by G.

61. A, B, and C agree to purchase a farm of 95 acres, for \$1250, of which C takes 12 acres at cost. A advances \$400, and B \$500, agreeing at the same time to share the 83 remaining acres in proportion to their advance. It is required to know how much C's part cost him, how much land A and B have, and the cost of their shares?—First, $\$1250\div95=\$13\frac{5}{19}$ per acre, and $13\frac{3}{19}\times12=\$157\frac{17}{19}$, cost of what C took at cost; then $500+400=900$, and $1250-900=\$350$, C advanced; then $350-157\frac{17}{19}=\$192\frac{2}{19}$, what C has left; and $192\frac{2}{19}+400+500=\$1092\frac{2}{19}$, what all have left; then $1092\frac{2}{19}\div83=\$12\frac{826}{1577}$ per acre, when 83 acres are to be divided according to A's and B's advance; then $13\frac{3}{19}-12\frac{826}{1577}=\frac{1037}{1577}$, and $\frac{1030}{1577}\times12=\$7\frac{1321}{1577}$ which is to be divided in proportion to what each has left before the addition of what is gained by the 12 acres, taken at cost.

Thus, $1092\frac{2}{19} : 7\frac{1321}{1577} :: 192\frac{2}{19} : \$1\frac{4113895}{10863953}$, C's part of the gain;
" : " :: 400 : $\$2\frac{9462846}{10863953}$, A's " " " "
" : " :: 500 : $\$3\frac{6386581}{10863953}$, B's " " " "

Then, $400+2\frac{9462846}{10863953}=\$402\frac{9462846}{10863953}$, A's money after the addition of his gain; and $500+3\frac{6386581}{10863953}=\$503\frac{6386581}{10863953}$, B's money after the addition of his part of the gain; also $192\frac{2}{19}+1\frac{4113895}{10863953}=\$193\frac{5213895}{10863953}$, C's money, after the addition of his part of the gain. Then A's, B's, and C's money $=\$1099\frac{10199369}{10863953}$, also A's and B's, $=\$906\frac{4985474}{10863953}$. Then if $1099\frac{10199369}{10863953} : 83 :: 906\frac{4985474}{10863953} : 68\frac{12144054587}{2987145929}$ acres, A and B have. Ans. Then $400-2\frac{9462846}{10863953}=\$397\frac{1401107}{10863953}$, cost of A's share of the purchase; then $500-3\frac{6386581}{10863953}=\$496\frac{4477372}{10863953}$, cost of B's share of the purchase; and $7\frac{1321}{1577}-1\frac{4113895}{10863953}=\$6\frac{4986474}{10863953}$, to which add $\$350=\$356\frac{4986474}{10863953}$, what C's share cost him.

62. B and C bought 1200 acres of land at \$1 per acre, each paying \$600. Sometime after, C on viewing it, offers to take a certain square piece at \$1,75 per acre, to the amount of his advance, to which B consents. How many acres will each have, what is the length of each side of C's lot, and what does B's part cost him per acre?—First, if $\$1,75 : 1 :: \$600 : 342$ acres, 3 roods, $17\frac{1}{4}$ rods, C has, and 1200 acres, less C's part $=857$ acres, $22\frac{3}{4}$ rods B has; and if 857 acres, $22\frac{3}{4}$ rods : \$600 :: 1 acre : 70 cts. per acre B's; and $\sqrt[2]{342}$ acres, 3 roods, $17\frac{1}{4}$ rods $=234$ rods, 3 ft. $6\frac{3}{4}$ inches is the length of either side of C's lot.

63. Sold a piece of cloth for 850 guineas, and gained upon every yard $\frac{1}{8}$ the prime cost of an ell-English; Required the cost of the piece.

First, $\begin{matrix}1 \text{ ell Eng.} = 5qr. \\ 1 \text{ yard} \quad = 4qr. \times \frac{1}{8} = \frac{5}{32} + \frac{32}{32} = \frac{37}{32}\end{matrix}$, (i. e.) 750 guineas $= \frac{37}{32}$, and 750 guineas $=$ £$892\frac{1}{2}$. And if $37 : 892\frac{1}{2} :: 32 :$ £771, 17*s*. $10\frac{2}{37}$*d*. cost of the piece. Ans.

64. A person having spent in one year all his income and $\frac{1}{4}$ as much more, found that by saving $\frac{1}{15}$ of his income afterward, he could make good the deficiency in 4 years, and have $20 left. Required his income.—Now $\frac{1}{15}$ of his annual income for four years is $\frac{4}{15}$ of it for one year; and $\frac{4}{15}$ of one year's income is $20 more than $\frac{1}{4}$ of it, therefore $20 is $\frac{1}{60}$ of his income. Hence, if $60 : 20 :: \overline{60 \times 60} :$ $1200. Ans.

65. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had £537 left. What did his father bequeath him?—First, $\frac{16}{16}$ = the whole legacy; then if $\frac{16}{16} - \frac{5}{16} = \frac{11}{16}$, left at the end of 3 months: then $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{16} = \frac{165}{384}$, and $\frac{11}{16} - \frac{165}{384} = \frac{1584}{6144} =$ £537, and if $1584 : 537 :: 6144 :$ 2082*l*. 18*s*. $2\frac{2}{11}$*d*. Ans.

66. A gay young fellow soon got the better of $\frac{3}{7}$ of his fortune; he then gave £1500 for a commission, and his profusion continued till he had but £450 left, which he found to be just $\frac{3}{8}$ of his money, after the purchase of his commission; What was his fortune at first?—First, if $3 : 450 :: 8 : 1200$, and $1200 + 1500 =$ £$2700 = \frac{5}{7}$ of his fortune; then if $5 : 2700 :: 7 :$ £3780. Ans.

67. A merchant begins the world with £1500, and finds that by his distillery he clears £1500 in 7 years; by his navigation £1500 in 9 years; and that he spent in gaming £1500 in $3\frac{1}{2}$ years. How long will his estate last?

First, if $\left\{\begin{matrix} 7 & : 1500 :: 1 : \text{£}214\frac{2}{7} \\ 9 & : \quad \text{"} \quad :: 1 : 166\frac{2}{3} \\ 3\frac{1}{2} & : \quad \text{"} \quad :: 1 : 428\frac{4}{7} \end{matrix}\right\}$ Then, $\overline{428\frac{4}{7} - \overline{214\frac{2}{7} + 166\frac{2}{3}}} : 1 ;$ £1500 . $31\frac{1}{2}$ years. Ans.

68. A younger brother received £1560, which was just $\frac{7}{12}$ of his elder brother's fortune; and $5\frac{3}{8}$ times the elder brother's money is $1\frac{2}{3}$ the value of the father's estate. What was the father worth?—First, if $7 : 1560 :: 12 :$ £$2674\frac{2}{7}$, the elder's fortune; and if $1\frac{2}{3} : 2674\frac{2}{7} :: 5\frac{3}{8} :$ £8624, 11*s*. $5\frac{1}{7}$*d*. Ans.

69. A gentlemen divided his fortune among his sons, giving A £9 as often as B £5, and to C but £3 as often as to B £7; C's dividend was £$1537\frac{5}{8}$. What did the whole estate amount to?—First, as $7 : 5 :: 3 : 2\frac{1}{7}$; then if $2\frac{1}{7} : 1537\frac{5}{8} :: \overline{9 + 5 + 2\frac{1}{7}} :$ £11583, 8*s*. 10*d*. Ans.

70. A, paid $50 for linen and cotton cloth, paying a dollar for 3 yards of linen, the same for 5 yards of cotton; he afterwards sold $\frac{1}{4}$ of his linen and $\frac{1}{5}$ of his cotton for $12, which was 60

cents more than it cost; How much of each did he purchase?—First, $12—60 cts.=$11,40 cost of $\frac{1}{4}$, and $\frac{1}{5}$; then if $\frac{1}{4}+\frac{1}{5}$: 11,40 : : $\frac{4}{4}+\frac{4}{5}$: $45,60, cost of his linen and $\frac{4}{5}$ of his cotton; then $50—$45,60=$4,40, price of the cotton's $\frac{1}{5}$ part, hence $4,40×5=$22, paid for cotton; then $50—$22=$28, paid for linen; then $22×5=110 yards of cotton; then $28×3=84 yards of linen.

71. A gentleman had 7*l.* 17*s.* 6*d.*, to pay among his laborers; to every boy he paid 6 pence, to every woman 8 pence, and to every man 16 pence; there was 1 boy to 3 women, and 1 woman to 2 men; What was the number of each?

First, 1 boy + 3 women + 6 men = 10 persons,
6*d.* + 24*d.* + 96*d.* = 126*d.*,

Then, if 126*d.* : 10 : : £7, 17*s.* 6*d.* : 150 persons in all; then, if 10 : 150 : : 1 : 15 boys, then 15×3=45 women, then 45×2=90 men.

72. A, B, C, and D caught with their net 522 fish, of which A was to have a certain number; B was to have 12 more than A, C 7 less than B, and D as many as A and C. Required the share of each.—First, A=1 share; then B=1 share and 12 fish; then C=1 share and 5 fish; and D=2 shares and 5 fish, the sum of which is 5 shares and 22 fish; then 522—22=500=5 shares; 5 : 500 : : 2 : 200; then 200+5=205 fish, D has; then 500÷5=100 fish, A has; then 100+12=112 B has; then 112—7=105, C has.

73. A, with an intention to clear 30 guineas, (22*s.* 9*d* each,) on a bargain with B, rates hops at 16*d.* per lb. that cost him but 10*d.* per lb. B, apprized of this, set down malt, that cost him 10*s.* 10*d.* per barrel, at an adequate price. How much malt did they contract for?—First, if 10*d.* : 16 : : 10*s.* 10*d.* : 17$\frac{1}{3}$*s.*, then 17$\frac{1}{3}$*s.*—10*s.* 10*d.*=6$\frac{1}{2}$*s.* gain per barrel to be adequate to A's barter price, and as many barrels are contracted for as 6$\frac{1}{2}$*s.* is contained times in the 30 guineas; hence 30 guineas÷6$\frac{1}{2}$*s.*=105 barrels. Ans.

74. A man that was feeding his geese, was accosted by another with, Good morning, with your hundred geese. He replied, I have not a hundred, but if I had half as many more as I now have, and two geese and a half, I should have a hundred. How many had he?—First, 100—2$\frac{1}{2}$=97$\frac{1}{2}$, and 97$\frac{1}{2}$=3 halves, hence 3 : 97$\frac{1}{2}$: : 2 : 65 geese. Ans.

75. Three men met at an inn, and two of them brought provisions with them; but the third not having any, proposed to the other two that they should all eat together, and he would pay them for his proportion. This being agreed to, A produced five loaves, and B three loaves, which they eat together, and C paid 8 equal pieces of money as the value of his share; How

much of the pay should A and B severally have?—First, $3+5=8$ loaves; then if loaves $8:3$ men $::3$ loaves $:1\frac{1}{8}$, therefore B produced $\frac{1}{8}$ more than he eat; then $8\div3$ men$=2\frac{2}{3}$ loaves that each eat; hence $2\frac{2}{3}=\frac{8}{8}$, and $\frac{8}{8}-\frac{1}{8}=\frac{7}{8}$ that C eat belonged to A; and since C paid 8 pieces of money, and $\frac{7}{8}$ of what C eat belonged to A, it shows that 7 of the 8 pieces of money belonged to A, which leaves 1 piece for B.

76. If 20 feet of iron railing weigh 1000 lbs. when the bars are $1\frac{1}{4}$ inch square, what will 50 feet come to at $9\frac{3}{4}$ cts. per lb. when the bars are $\frac{7}{8}$ of an inch square?—First, $50=20\times2\frac{1}{2}$; then $1\frac{1}{4}\times8=10$, and $10\times10=100$, and $\frac{7}{8}\times\frac{7}{8}=49$; then, if $100:1000::49:490$ lb$=20$ ft of the $\frac{7}{8}$ inch square; then $490\times2\frac{1}{2}=1225$ lbs.; then 1225 lbs.$\times9\frac{3}{4}$ cts.$=\$119,4375$. Ans.

77. Bought a quantity of linen at 12*s.* 8*d.* for 3 yards, and by selling it at 32*s.* 8*d.* for 7 yards, I gained as much as 24 yards cost; Required the number of yards bought.—First, if $3:12s.::7:£1,9s.6\frac{2}{3}d.$; then $£1,12s.8d.-£1,9s.6\frac{2}{3}d.=3s.2\frac{1}{3}d.$ Then if $3s.2\frac{1}{3}d.:7::4s.2\frac{2}{3}d.\times24:224$ yards. Ans.

78. Laid out in a lot of muslin £500, upon examination of which 3 parts in 9 proved damaged, so that I could make but 5*s.* a yard of the same; and by so doing, find I lost £50 by it. At what rate per ell Eng. am I to part with the undamaged muslin, in order to gain £50 upon the whole?—First, if $3:£50::9:£150$ loss on £500, at £50 per 3 parts, then $£500-150=£350$ that 9 parts cost at 5*s.* a yard; then $£350\times20\div5=1400$ yards in all; then if $9:1400::3:466\frac{2}{3}$ yds. damaged, then $1400-466\frac{2}{3}=933\frac{1}{3}$ yards undamaged, and $933\frac{1}{3}\times4\div5=748\frac{4}{15}$ ells Eng.; Then $466\frac{2}{3}\times5s.=£116,13\frac{1}{3}s.$ cost of 3 parts at 5*s.* per yard; then $£550-£116,13\frac{1}{3}s.=£433,6\frac{2}{3}s.$ cost of the 6 undamaged parts, then $£433,6\frac{2}{3}s.\div748\frac{4}{15}=11s.7\frac{2}{7}d.$ per ell Eng. Ans

79. A has £100 of B's money, for the remittance of which B allows him 9 per cent. What sum of money must he remit to discharge himself of the £100?—First, as $\overline{100+9}:100::100:£91\frac{81}{109}$ to be remitted, and $£100-£91\frac{81}{109}=£8\frac{28}{109}$ his commission.

80. In a series of proportionals, the first is 4, the third is 12, and the product of the second and third is 112,8. What is the diff. of the 2d and 4th?—First, $112,8\div12=9,4$ the second. As $4:9,4::12:28,2$, and $28,2-9,4=18,8$. Ans.

81. A shepherd being asked how many sheep he had, said, If I had as many, half as many, and $\frac{1}{4}$ as many more, I should have 264; How many had he?—First, what he had$=\frac{4}{4}$, then as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, is $\frac{7}{4}$, and $\frac{4}{4}+\frac{7}{4}=\frac{11}{4}$, and $\frac{11}{4}=264$, then if $11:264::4:96$ sheep. Ans.

82. B delivers to C $1200, to be invested in trade, on condition that if C added $500 to it. and gave his time as manager, he should have $\frac{2}{5}$ of the gain; What was C's time valued at?—First, if C's time and $500 was equal to $\frac{2}{5}$, 1200=\frac{3}{5}$; hence, if 3 : 1200 : : 2 : $800, and $800—$500=$300. Ans.

83. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$ of a £, What will $\frac{8}{15}$ of an ell Eng. cost?—First, $\frac{3}{5}$ of a yard=$\frac{3}{5}$ of $\frac{4}{1}$ of $\frac{1}{5}$=$\frac{3\times4\times1}{5\times1\times5}$=$\frac{12}{25}$ of an ell. Then if $\frac{12}{25}$: $\frac{7}{12}$ :: $\frac{8}{15}$; Operation, $\frac{25\times7\times8}{12\times12\times15}$=$\frac{35}{72}$£,=9*s*. 8$\frac{2}{3}$*d*. Ans.

84. A man dying left his wife in expectatation that a child would be afterwards added to the family, and in making his will ordered that if the child were a son, $\frac{2}{3}$ of his estate should belong to him, and the remainder to his mother; but if it were a daughter, he appointed the mother $\frac{2}{3}$, and the child the remainder; now it happened that the addition was both a son and a daughter; by which the widow lost in equity $2400 more than if there had been only a girl. What would have been her dowry, had she had only a son?—Now it is evident that each one's part suffers a loss, for there were more heirs than expected; and the son has $\frac{2}{3}$, and the mother $\frac{1}{3}$, hence 3 shares in all, which are reduced to 4 parts; and it is evident that the son's part is what the mother and daughter do not have of the estate, and if the mother suffers in equity $2400, the daughter suffers in proportion; hence if 2 : 2400 : : 1 : 1200, then 2400+1200=$3600 =the son's part, and if 4 parts are condensed in 3 shares, the son's part suffers in proportion;

Hence, if 4 : 3 : : 3 : 2$\frac{1}{4}$, And, if 1$\frac{1}{2}$: 3600 : : 2$\frac{1}{4}$: 5400
And if 4 : 3 : : 2 : 1$\frac{1}{2}$, Or, of 1$\frac{1}{2}$: " : : 3 : 7200

2)12600

The value of the estate=3 shares=$6300
And if 3 : 6300 : : 2 : 4200, then 6300—4200=$2100. Ans.

85. A man lays out 30 cents for apples and pears, buying his apples at 4, and his pears at 5 for a cent, and afterwards sold $\frac{1}{2}$ of his apples, and $\frac{1}{3}$ of his pears for 13 cents which was the prime cost; I demand the number he bought of each?—If $\overline{\frac{1}{2}+\frac{1}{3}}$: 13 : : $\overline{\frac{1}{2}+\frac{1}{3}}\times2$: 26 cts., then 30—26=4 cts. that $\frac{1}{3}$ of the pears cost, then $\frac{3}{3}$=12 cents he paid for pears, then 12×5=60 pears bought, then 30—12=18, and 18×4=72 apples.

86. A person found two sums of money; $\frac{1}{4}$ of the first added to $\frac{1}{5}$ of the second was $120. The two sums together were $400; What was each sum?—First, $\overline{\frac{1}{4}+\frac{1}{5}}$: 120 : : $\overline{\frac{3}{4}+\frac{3}{5}}$: $360, 400—360=$40=$\frac{1}{4}$ of the first sum, then 40×4=$160, the first sum, and 400—160=$240, the second sum.

87. If A can do as much work in 3 days as B can do in 4$\frac{1}{2}$ days, and B as much in 9 days as C in 12 days, and C as much

in 10 days as D in 8 days, how many days' work of D are equal to 5 days' work of A?

First, $\begin{array}{l} 3\times 9\times 10=270, \\ 4\frac{1}{3}\times 12\times 8=432, \end{array}$ then 270 : 432 : : 5 : 8 days. Ans.

88. If 25 pears can be bought for 10 lemons, and 28 lemons for 18 pomegranates, and 1 pomegranate for 48 almonds, and 50 almonds for 70 chestnuts, and 108 chestnuts for $2\frac{1}{2}$ cts., how many pears can I buy for $1,35 cts.?

First, $\begin{array}{l} 25\times 28\times 1\times 50\times 108=3780000, \\ 10\times 18\times 48\times 70\times 2\frac{1}{2}=1512000, \end{array}$

then, 1512000 : 3780000 : : 1 ,35 : $337\frac{1}{2}$. Ans.

89. If 12 oxen eat up $3\frac{1}{2}$ acres of grass in 4 weeks, and 21 oxen eat up 10 acres in 9 weeks, how many oxen would eat up 24 acres in 18 weeks; the grass being at first equal on every acre, and growing uniformly?—A part of the given number of oxen, in each condition of this question, must be supported by the grass at first standing on the given number of acres, and the remaining part must be supported by the growth; and the number of oxen that can be supported by the grass at first standing on the ground, must be in a direct ratio to the number of acres, and in an inverse ratio to the time of grazing; and the number of oxen that can be supported by the growth of the grass, must be in a direct ratio to the number of acres, without any regard to the time of grazing, for it is evident that the number of oxen that would consume the growth of any given number of acres during any given time would consume the same growth continually. By the first condition of the question, if $\overset{A.}{3\frac{1}{2}} : \overset{O.}{12} : : \overset{A.}{10} : 34\frac{2}{7}$ oxen to consume 10 acres of grass and its growth in 4 weeks, but since only 4 weeks are considered in the preceding proportional statement, and 5 of the 9 weeks are yet to pass, and since it requires the grass at first and its growth to support $34\frac{2}{7}$ oxen only 4 weeks, it shows that $34\frac{2}{7}$ oxen cannot be supported by 10 acres of grass and its growth for 9 weeks, hence why the number of oxen that can be supported by the grass at first standing on the ground, must be in an inverse ratio to the time of grazing; therefore, if $\overset{W.}{4} : \overset{O.}{34\frac{2}{7}} : : \overset{W.}{9} : 15\frac{5}{21}$ oxen to consume the grass at first and its growth, on 10 acres in 4 weeks; then 21 oxen less $15\frac{5}{21}$ are $5\frac{16}{21}$ oxen which in 9 weeks would consume the growth of 10 acres of grass during the 5 remaining weeks, and to consume the growth of 10 acres of grass for 9 weeks, it requires $\frac{9}{5}$ as many oxen, hence if $\overset{W.}{5} : \overset{O.}{5\frac{16}{21}} : : \overset{W.}{9} : 10\frac{13}{35}$ oxen would consume the growth of 10 acres of grass during 9 weeks, and 21 oxen less $10\frac{13}{35}$ oxen are $10\frac{22}{35}$ oxen would consume the grass at first on 10 acres *in 9 weeks, and if* $\overset{A.}{10} : \overset{O.}{10\frac{22}{35}} : : \overset{A.}{24} : 25\frac{39}{175}$ oxen to consume the

grass at first on the 24 acres, in 9 weeks, but since the oxen must be supported 18 weeks, say if $\overset{\text{W.}}{9} : \overset{\text{O.}}{25\tfrac{89}{175}} :: \overset{\text{W.}}{18} : 12\tfrac{132}{175}$ oxen would eat up the grass at first on the 24 acres, and since the number of oxen that would consume the growth of any given number of acres during any given time, would consume the same growth continually, and since $10\tfrac{13}{35}$ oxen would consume the growth of 10 acres of grass in 9 weeks, say if $\overset{\text{A.}}{10} : \overset{\text{O.}}{10\tfrac{13}{35}} :: \overset{\text{A.}}{24} : 24\tfrac{156}{175}$ oxen would consume the growth of 24 acres; and $24\tfrac{156}{175}$ oxen consume the growth, and $12\tfrac{132}{175}$ oxen the grass at first standing on the 24 acres, the sum of which is $37\tfrac{143}{175}$ oxen, and the answer required.

PERCENTAGE PROPORTIONALS,

AND SUCH INTRICATE ARITHMETICAL QUESTIONS AS OFTEN OCCUR IN SCHOOLS; RANKED IN CLASSES ACCORDING TO THEIR NATURE.

CLASS I,

is when it is required to find the gain or loss per cent.

EXAMPLES.

1. G bought of W a lot of lamb's wool washed, weighing 2720 lbs. at $46\tfrac{1}{2}$ cents per lb.; but thinking it was not sufficiently dry, he takes it only on condition that a parcel be dried, and by that the state of the whole be determined. On trial the loss was ascertained, and deduction made as stipulated, when G paid W \$1169,94 in full; Required the percentage loss by drying?—First, $2720 \times 46\tfrac{1}{2} = \$1264,80$, its cost, then $1264,80 - 1169,94 = \$94,86$, its loss; then, if $1264,80 : 94,86 :: 100 : 7\tfrac{1}{2}$ per cent. Ans.

2. A stationer sold quills at $\$1,83\tfrac{1}{3}$ cts. per thousand, by which he cleared $\tfrac{3}{8}$ of the money; but as they grew scarcer he raised them to \$2,25 cts. per thou.; What did he clear per cent. by the latter price?—First, $\overline{183\tfrac{1}{3} \times 3} \div 8 = 68\tfrac{3}{4}$ cts. first gain, then $183\tfrac{1}{3} - 68\tfrac{3}{4} = \$1,14\tfrac{7}{12}$, first cost; then $225 - 114\tfrac{7}{12} = \$1,10\tfrac{5}{12}$, last gain per thousand; Then $\$1,14\tfrac{7}{12} : \$1,10\tfrac{5}{12} :: \$100 : \$96,36\tfrac{4}{7}$ cts. Ans.

3, What would be the duty on a piece of flannel 30 yards long and 42 inches wide, if it was estimated at 40 cents per square yard, and 30 per cent. ad-valorem, and what percentage would it pay on the original cost, if charged at 15*d.* sterling per yard?—First, 15*d.* sterling=\$0,278, and 30 yards long and 42 inches wide=35 square yards of flannel; then $\$0,278 \times 35 = \$9,73$, its extra cost, and cents 40×35 yds.=\$14 its cost, and

14+9,73=$23,73, and $23,73÷35=$0,6762; Then if ,40: ,30: : ,6762 : 50 per cent. Ans. 2. Then 30 yards×3 ft.×12 in. =1080 in. which×42 in.=45360, and 3×3×144=1296 in. in 1 yd.; then 45360÷1296=35 yds. which multiplied by 40 cts. =$14=first cost of the flannel; then if $100:30::$14:$4,20. Ans. 1.

4. A bought flour for cash, and sold it to B at an advance. B sold it to C at 10 per cent. advance, and C, on selling it to D, gained $71,28, equal to 12 per cent. profit; which was 4 per cent. more than A made, though he bought it at $5 per barrel. Required B's gain, how much C received, and the number of barrels in the lot?—Now since C gained 12 per cent. which was 4 per cent. more than A made, though he bought it at $5,00 per barrel, it shows that A gained 8 per cent., and sold it at at $5,40 per barrel to B, and B sold it to C at 10 per cent. profit, which caused the flour to stand C $5,94 per barrel. Then, if $1,00: ,12 : : $5,94 : $0,7128=C's gain per barrel on selling it to D. And if $0,7128 : 1 : : $71,28 : 100 barrels. Ans. 3. And since it cost C $5,94, and he gained $0,7128, it stood D $0,7128+$5,94=$6,6528 per barrel; and $665,28 per 100 barrels. Ans. 2. And since it stood B $5,40, and C $5,94, say 5,94—5,40=54 cents, B gained per barrel; hence $54 per hundred barrels. Ans. 1.

5. A merchant bought a quantity of goods for $734, and sold them for $888,14. How much did he gain per cent?—If $734 : $888,14 : : $100 : $121, then 121—100=21 per cent. Ans.

6. Bought 30 hogsheads of molasses for $600; paid in duties $20,66; for freight $40,78; for portage $6,05; and for insurance $30,84. If I sell it for $26 per hogshead, how much shall I gain per cent.?—First, 600+20,66+40,78+6,05+30,84= $698,33, and 698,33÷30=$23,27766 per hhd. Then, if 23,27766:26::100:$111,699, and 111,699—100=$11,699. Ans.

7. A guardian paid his ward $3500 for $2500 which he had in his hands 8 years. What rate of interest did he allow?—Thus, if 2500 : 3500 : : 100 : $140, and 140—100=40, and 40 ÷8=5 per cent. Ans.

8. At what rate per cent. will $950,75 amount to $1235,975 in 5 years?—If 950,75 : 1235,975 : : 100 : 130, and 130—100 =$30 for 5 years; then 30÷5=6 per cent. Ans.

9. A house completely finished, cost the owner $12894; it is 4 stories high, and the ground floor is divided into two shops, one of which is let at $225, the other at $200 a year; the three upper stories are let for $450 a year; the annual expense for repairs is $36,89. What per cent. does the house pay?—First, 225+200+450=$875, and 875—36,89=$838,11. And, if $12894 : $838,11 : : $100 : $6,50=6½ per cent. Ans.

CLASS II,

Requires to know how an article must be sold to gain or lose so much per cent.

EXAMPLES.

1. Bought a pipe of wine containing 126 gallons, at 10*s.* per gallon, but by accident 16 gallons leaked out; At what rate must the remainder be sold per gallon, to gain on the whole prime cost, at the rate of $12\frac{1}{2}$ per cent.?—First, 126 gallons worth £63; and 126—16=110 gallons not leaked out; and, if 100 : 112,5 : : 63 : £70,875, and 70,875÷110=12*s.* $10\frac{7}{11}$*d.* per gal. Ans.

2. A merchant bought a quantity of goods in Boston, for $500, and paid $43 for their transportation; he then sold them so as to gain 24 per cent on the whole cost; For how much did he sell them?—$500+$43=$543; then, if 100 : 124 : : 543 : $673,32. Ans.

3. A, of Providence, has in his hands $500 due to G, of Charleston, for net proceeds of his cotton; this is remitted to G per bill on M in his favor, when bills on Charleston are 2 per cent. discount. Required the amount of the bill?—Thus, if 100 : 102 : : 500 : $510. Ans.

4. At what price per quintal should fish be rated, so as to give 9 months credit, and allow 5 per cent discount for present payment, and gain 15 per cent. when it is now worth in Boston 15*s.* per quintal?—If 100 : 115 : : 15 : 17,25*s.*=15*s.*+15 per cent., and £3,75=interest of £100 for 9 months, then, if 100 : 3,75 : : 17,25 : ,6234 *s.*=discount of 17,25 *s.* and 17,25+,6234=17,8734*s.* Ans.

5. A person lent his friend a sum of money, to receive interest for the same at 6 per cent. per annum, simple interest; at the end of 3 years he received for principal and interest, $383,5. What was the sum lent?—The interest of $100 for 3 years at 6 per cent. is $18, then if 118 : 100 : : 383,5 : $325. Ans.

6. A merchant imported 10 tons of iron at $95 per ton; the freight and duties amounted to $145, and other charges to $25. At what price per lb. must he sell it to gain 20 per cent?—First, $\overline{95\times10}+145+25$=$1120, and 10 tons=22400 lbs. Then if 100:120::1120:$1344, and $1344÷22400=6 cts. Ans.

CLASS III,

Requires the cost of an article, when there is gained or lost so much per cent.

EXAMPLES.

1. Three parcels of beef, of 60 barrels each, were received at *Baltimore* from Boston, marked W, X, Y. The lot marked

W was found to be 50 per cent. better than the others. If the whole sold at $10 per barrel, how must the sales be adjusted between the owners of the beef?—60×3=180 barrels in all; one parcel 100, another 100, and another $\overline{100+50}$=150, and 100+100+150=350; then 350 : 300 : : 100 : $8,57$\frac{1}{7}$×60=$514,28$\frac{4}{7}$, Y's or X's. And if 350 : 300 : : 150 : $12,85$\frac{4}{7}$×60=$771,42$\frac{4}{7}$, W's.

2. A and B cleared by an adventure at sea 45 guineas, which was 35 per cent. upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; How much did each adventure?—Each guinea=£1$\frac{2}{5}$, and 1$\frac{2}{5}$×45=£63. Then, if 35 : 63 : : 100 : £180 adventured; and $\overline{11+8}$: 180 : : 11 : £75, 15*s.* 9$\frac{9}{19}$*d.* A adventured; and 19 : 180 : : 8 : £104, 4*s.* 2$\frac{10}{19}$ B adventured.

3. A father left two sons (the one 11, the other 16 years of age) $10000, to be divided so that each share, being put to interest at 5 per cent. might amount to equal sums when they would be respectively 21 years of age. Required the share of each.—First, the share of the younger son must be on interest 10 years, at 5 per cent., and that of the older 5 years. And the amount of $1 for 10 years, at 5 per cent. is $1,50; and the amount of $1 for 5 years at 5 per cent. is $1,25; then $\overline{1,50+1,25}$: 10000 : : 1,50 : 5454\frac{5}{11}$, the older's share; and 2,75 : 10000 : : 1,25 : 4545\frac{5}{11}$, the younger's share.

4. A purchased a house for $1575, and repaired it for D, who agreed to pay him a rent of $220 per annum, which was 12 per cent. for his money that he paid for the house and its repair. What was the cost of repairing it?—If 12 : 100 : : 220 : 1833\frac{1}{3}$, and 1833$\frac{1}{3}$—1575=258$\frac{1}{3}$. Ans.

5. A had 12 pipes of wine, which he parted with to B at 4$\frac{1}{2}$ per cent. profit, who sold them to C for $40,60 advantage; C made them over to D for $605,5, and cleared thereby 6 per cent. How much a gallon did this wine cost?—12 pipes=1512 gallons. If 106 : 100 : : 605,5 : $571,22641,—40,60=$530,62641; and if 104$\frac{1}{2}$: 100 : : 530,62641 : $510,12154,÷1512=33$\frac{1116}{1512}$ cents. Ans.

6. A's commission, at 5 per cent. on a consignment of coffee, was $47,5, by the gross sales of which the shipper made 25 per cent profit. What was it invoiced at?—If 5 : 47,5 : : 25 : $237,5,—47,5=$190, then 25 : 237,5 : : 100 : $950,—190=$760. Ans.

7. *G owns* $\frac{2}{7}$ *of* $\frac{1}{2}$, and $\frac{3}{7}$ of $\frac{4}{11}$ of the other $\frac{1}{2}$ of an undivided *estate; suppose this* estate rents for $1556,94, being equal to 6 *per cent. per annum* on its value, and that he sells $\frac{1}{3}$ of his part

on such terms as to yield the purchaser 8 per cent. per annum on his payment; How much does G receive, and what share has he now in the estate?—First $\frac{2}{7}$ of $\frac{1}{2}=\frac{2}{14}=\frac{1}{7}$, and $\frac{3}{7}$ of $\frac{4}{11}$ of $\frac{1}{2}=\frac{12}{154}$, $+\frac{1}{7}=\frac{34}{154}$, $\div 2=\frac{17}{154}$, G owns of the estate. And if 154 : 1556,94 : : 17 : $171,87, and if 8 : 100 : : 171,87 : $2148,37½, G receives.

8. A, on preparing for a voyage to Calcutta, purchased of G, specie dollars. to be paid in 18 months with interest. Supposing the premium on the dollars to be 3 per cent. and that G would have 5 per cent. per annum for the use of his money to be inserted in the note, which was given for $22145; I would know the sum purchased.—The interest of 3 per cent. at 5 per cent. for 18 months is ,225, and the interest of $100 for 18 months, at 5 per cent. is $7,5, and $\overline{100+7,5+3+,225}$: 100 : : 22145 : $20000. Ans.

9. B owns $\frac{3}{7}$ of a ship which cost him $3691, and sells $\frac{3}{8}$ of his part to C at 12 per cent. advance; Required the share of each in the ship, the cost of C's part, and how much B's part stands him?—If 8 : 3691 : : 3 : $1384,125=$\frac{3}{8}$ of $3691, and if 100 : 112 : : 1384,125 : $1550,22, C's part cost him. Ans. And 1550,22—1384,125=166,095, and 3691—166,095=$3524,905, B's part stands him. Ans. And C's part is equal to $\frac{3}{8}$ of $\frac{3}{7}=\frac{9}{56}$. Ans. And B's part is equal to $\frac{5}{8}$ of $\frac{3}{7}=\frac{15}{56}$. Ans.

10. The interest of a certain sum at simple interest for 16 years, at 5 per cent. per annum, remaining unpaid, wanted but $9,32 of the principal. What was the sum?—$100 on interest for 16 years, at 5 per cent. is equal to $180, and $200—180=$20; then if 20 : 100 : : 9,32 : $46,60. Ans.

11. A purchased at public sale $\frac{25}{37}$ of a store, and soon after he paid $1230,075 for $\frac{7}{37}$ of the same, which was at 10 per cent. advance on the rate of the first purchase; and to own the whole he bought the remaining part at 25 per cent. advance on it. What did the store cost him?—If 110 : 100 : : 1230,075 : $1118,25, first purchase cost of 7 shares; then if 7.1118,25::25:$3993,75, cost of 25 shares; then if 7 : 1118,25 : : 5 : $798,75, first purchase cost of 5 shares, and if 100 : 125 : : 798,75 : $998,4375, cost of 5 shares, then 3993,75+1230,075+998,4375=$6222,2625. Ans.

CLASS IV,

Requires to know what would be gained or lost per cent. by goods sold at another rate, when there is gained or lost so much per cent. by being sold at a certain rate.

EXAMPLES.

1. If by selling goods at 2*s.* 3*d.* per lb. I clear cent. per cent., what do I clear per cent. by selling them at 9 guineas per cwt.?

—2*s.* 3*d.*=27*d.*, and 9 guineas=2268*d.*; 2268÷112=20¼*d.* per lb.; then if 27 : 200 : : 20¼ : 150, and 150—100=50 per cent. Ans.

2. If by selling goods at $2½ per cwt. I gain 20 per cent., what do I gain or lose per cent. by selling at $2¼ per cwt.?—If 2,5 : 120 : : 2,25 : 108, and 108—100=8 per cent. gain. Ans.

3. I sold a horse for $60, and by so doing lost 20 per cent., whereas I ought to have cleared 30 per cent.; What was the real value of the horse?—$\overline{100—20}$: 60 : : 100 : $75, and if 100 : 130 : : 75 : $97,50. Ans.

4. If wine, sold at $1,50 per gallon, be 15 per cent. profit, what would be the gain or loss per cent. if sold at $1,25 per gallon?—115 : 100 : : 150 : 130$\frac{10}{23}$ cts. first cost, and 130$\frac{10}{23}$—125 (the other rate,)=5$\frac{10}{23}$, loss on the first cost at the other rate, then if 130$\frac{10}{23}$: 5$\frac{10}{23}$: : 100 : $4,16$\frac{2}{3}$ loss. Ans.

5. A merchant purchased goods to the amount of $3472, which he sold at a loss of 12½ per cent. and invested the proceeds of the sale in other goods, which he sold at a profit of 13 per cent. Did he gain or lose by these transactions, and how much per cent.?—100 : $\overline{100—12,5}$: : 3472 : $3038 left, and if 100 : 113 : : 3038 : $3432,94, and 3472—3432,94=$39,06, lost, and consequently 1$\frac{31}{248}$ per cent. Ans.

CLASS V,

Requires an explanation of questions in Partnership, Barter, Commission, Brokerage, Insurance, and Discount.

EXAMPLES IN PARTNERSHIP.

1. A and B companied; A put in £45, and took ⅔ of the gain. What did B put in?—Thus, 3 : 45 : : 2 : £30. Ans.

2. A and B commenced business with equal sums of money. A gained a sum equal to ⅕ of his stock, but B lost $200, and then had only half as much as A; What was the original stock of each?—The stock of each=⅕; ⅕ of eithers stock and $200 =¼ as much as A's stock and gain; and A's stock=⅕, and his gain=⅕, the sum of which is ⅖; and B's stock less $200=⅗=½ of ⅖, hence $200=⅖ of either's stock, and consequently $500 either's original stock.

3. A, B, C, and D companied, and gained a sum of money, of which A, B, and C took $300; B, C, and D $400; C, D, and A $380; and D, A, and B $360; What was the gain o each?—300+400+380+360=1440, and 1440÷3=480, and 480—400=$80, A's gain; 480—380=$100, B's gain; 480=360=$120, C's gain; and 480—300=$180, D's gain.

4. *A and B purchased* 20 acres and 72 rods of land at $25,57 *per acre. By agreement*, A was to have 6 acres and 12 rods of

the land, and B the remainder, and to pay $4 per acre more than A. How much did each man's land cost him per acre?—$25,57 ×20,45 acres=$522,906 cost of the land; and acres 20,45—6,075=14,375 acres belong to B, and 14,375×$4=$57,50, extra price of B's land, then 522,906—57,5=$465,406+, and 465,406÷20,45=$22,758, A's land cost him per acre; and 22,758+4=$26,758 B's land cost him per acre.

5. Three persons purchase a vessel in company, towards the payment of which A advanced $\frac{2}{5}$, B $\frac{3}{7}$, and C £256; what did A and B pay, each, and what part of the vessel had C?

$\frac{2}{5}+\frac{3}{7}=\frac{14 \text{ plus } 15}{35}=\frac{29}{35}$, and $\frac{35}{35}-\frac{29}{35}=\frac{6}{35}$, C's part of the vessel.

As, $\frac{6}{35} : \frac{256}{1} :: \begin{cases} \frac{14}{35} : £597, 6s. 8d. \text{ A advanced;} \\ \frac{15}{35} : £640, \text{ B advanced.} \end{cases}$

6. A certain club spent at a supper, $43,56, and the expense of each was as many cents as there were persons in the company; what did each pay?—Thus, $\sqrt[2]{4356}=66$ cts. each. Ans.

7. A gentleman died, leaving three sons and a daughter, to whom he bequeathed his estate in the following manner—To the eldest son he gave 312 moidores, to the second 312 guineas, to the third 312 pistoles, and to the daughter 312 dollars; but when his debts were paid, there were but 312 johannes left; what must each have in proportion to the legacies bequeathed to each?

Moidores 312×$6,00 =1872
Guineas 312×$4,66$\frac{2}{3}$=1456
Pistoles 312×$3,66$\frac{2}{3}$=1144
Dollars 312×$1,00 = 312

Their sum,=$4784
And 312 johannes=$2496

And if 4784:2496::312:$162,78+,=£48, 16*s.* 8$\frac{1}{4}$*d.* daughter's;
4784 : 2496 : : 1872 : $.....=£293, 0*s.* 2*d.* first son's share;
" : " : : 1456 : $.....=£227, 17*s.* 10$\frac{3}{4}$*d.* 2d son's share;
" : " : : 1144 : $.....=£179, 1*s.* 2$\frac{1}{2}$*d.* 3d son's share.

8. A, B, and C put in £720, and gained £540, of which so often as A took £3, B took 5, and C 7. What did each put in and gain?—3+5+7)720(48 multiplicand to find the stocks, and 15)540(36 multiplicand to find the gains.

Then, $\begin{cases} 48\times3=£144 \text{ A's stock,} \\ 48\times5=£240 \text{ B's "} \\ 48\times7=£336 \text{ C's "} \end{cases}$ and $\begin{cases} 36\times3=£108 \text{ A's gain,} \\ 36\times5=£180 \text{ B's "} \\ 36\times7=£252 \text{ C's "} \end{cases}$

9. A and B commence trade in company; A advanced at first $500, and B $750; at the end of 4 months A adds $600 to his stock. What sum must B advance at the same time to entitle him at the end of the year to equal profits with A?

First, $\begin{cases} 600\times 8=4800 \\ 500\times 12=6000 \end{cases}$ And 750×12=9000; and 6000+4800 =10800, and 10800—9000=1800;

Then, if $\left\{\begin{matrix}10800\\8\end{matrix}\right\}$: 1800 : : $\left\{\begin{matrix}9000\\12\end{matrix}\right\}$: \$225. Ans.

10. A clears \$13 in 6 months, B \$18 in 5; and C \$23 in 9, his stock being \$72½; what then is the general stock?

First, \$72,5 : 9 : : 23 | \$72,50=C's stock,
5 : 18 | : \$102,13$\frac{1}{23}$ B's "
Then, 102,13$\frac{1}{23}$: 5 : : 18 | : \$61,46$\frac{7982}{10800}$ A's "
6 : 13 | \$236,09$\frac{97193}{124200}$. Ans.

11. A, B, and C companied; A put in the first of March, £30; B, the first of May, put in 80 yards of broadcloth; and the first of June C put in \$120. On the first of January following, they reckoned their gains, of which A and B took £228, B and C £215, 10*s*. and C and A £187, 10*s*. What was the whole gain, and the gain of each? what did they value a yard of cloth at? and what was C's dollar worth?—£288+£215, 10*s*.+£187, 10*s*. =£631, and 631÷2=£315, 10*s*. the whole gain; then £315, 10*s*.—£288=£87, 10*s*. C's gain; £315, 10*s*.—£215, 10*s*.= £100, A's gain, and £315, 10*s*.—£187, 10*s*.=£128, B's gain. To find the value of one yard of cloth, say, as £100, A's gain : £30 his stock : : £128 B's gain : £38, 8*s*. then inversely, As 10 months : £38, 8*s*. : : 8 months : £48 the value of the whole cloth. Then, As 80 yds. : £48 : : 1 yd. : 12*s*. Ans.; and As £100 A's gain : £30 his stock : : £87, 10*s*. C's gain : £26, 5*s*. then, inversely, As 10 months : £26, 5*s*. : : 7 months : £37, 10*s*. =\$120, then £37, 10*s*.=750*s*., and 750÷120=75*d*. Ans.

12. A, B, and C companied, and put in together £1911. A's money was in 3 months, B's 5 months, and C's 7 months; they gained £117, which was so divided as that ½ of A's gain was equal to ⅓ of B's, and ¼ of C's gain. What did each gain and put in?—Suppose A's gain was £2, then B's must have been £3, and C £4, by the question.

Then, as $\overline{2+3+4}$: £117 the whole gain : : $\left\{\begin{matrix}2 : £26, \text{ A's gain,}\\3 : £39, \text{ B's "}\\4 : £52, \text{ C's "}\end{matrix}\right.$

Then divide each man's gain by his time; and as the sum of these quotients is to each particular quotient, so is the whole stock to each man's particular stock. A's stock £693, 2s. 2d.; B's £623, 15s. 11½d.; C's £594, 1s. 10¼d. Ans.

EXAMPLES IN BARTER.

1. G received from H 760 lbs. of rough tallow to try out, at 60 cts. per 100 lb. clear, and was to take his pay in rough tallow at 8 cents per lb.; G returned 615 lbs. clear, and H paid the *balance due to G in* rough tallow. Allowing 18 per cent. for *waste what was the* balance due to G?—Thus, $\overline{100-18}$: 100

: : 615 . 750, then 760—750=10 lbs. of rough yet untried out; Then 100 : 60 : : 615 : $3,69 which will pay for $46\frac{1}{3}$ lbs. rough, which, less the 10 lbs: that remains in G's hands, shows the balance due to G is $36\frac{1}{3}$ lbs.

2. A has a chest of tea, weighing $3\frac{1}{2}$ cwt. the prime cost of which is £60; now allowing interest at 6 per cent. per annum, how must he rate it per lb. to B, so that by taking his note of hand, payable at 6 months, he may clear $50 by the bargain?—The interest=£2, 5s. and $50=£15; then $3\frac{1}{2}$ cwt.÷$\overline{60+15+2\frac{1}{4}}$ =3s. $11\frac{29}{38}$d. Ans.

3. A merchant sold a parcel of coffee at 15 cents per lb. and lost 10 per cent.; some time after, he sold another parcel of the same, to the amount of $700, and gained 40 per cent.; how many lbs. were there in the last parcel, and at what rate was it sold?—$\overline{100-10}$: 100 : : 15 : $16\frac{2}{3}$ cts. cost per lb., then 100 : $\overline{100+40}$: : $16\frac{2}{3}$: $23\frac{1}{3}$ cts. per lb. rate sold at to gain 40 per cent.; and $700÷$23\frac{1}{3}$ cts.=3000 lbs. in the last parcel.

4. B bought A's drove of swine, giving 18s. for hogs, 16s. for sows, and 2s. for pigs; there were as many hogs as sows, and for every sow there were 3 pigs, worth in all £50.—How many was there of each?—18s.+16s.+6s.=40s. that 5 swine sold for, then if 40s. : 5 : : £50 : 125 swine in all; and if shares 5 : 125 : : 3 : 75 pigs; then 125—75=50 hogs and sows, and 50÷2=25 of either.

5. A paid $100 for 100 animals, consisting of oxen, sheep, and geese, paying $10 for an ox, $1 for a sheep, and 1s. for a goose. How many of each did he buy?

First; 6 $\left\{\begin{matrix}60\\1\end{matrix}\right]$ 5 oxen, 54 geese Then, $\overline{100-54+5}$=41 sheep.

6. A has linen cloth at 30 cts. per yard, ready money, in barter 36 cents; B has 3610 yards of ribband at 22 cents per yard, ready money, and would have of A $200 in ready money, and the rest in linen cloth. What rate does the ribband bear in barter per yard, and how much linen must A give B? First, 30 : 36 : : 22 : $0,264 rate of ribband, then 20000÷22=$909\frac{1}{11}$, and 3610—$909\frac{1}{11}$=$2701\frac{10}{11}$, which ×,264=713,088, which÷,36= $1980\frac{13}{15}$ yards. Ans.

7. Jackson has 100 reams of paper at $$133\frac{1}{3}$$ ready money, in barter $$1,66\frac{2}{3}$$; Howard has pamphlets at $8\frac{1}{3}$ cts. apiece ready money, which he adequately charges, and insists on having the money for $\frac{1}{4}$ of the price of those he parts with. What number of books does J. receive, and how much money does H. receive, and what is his gain?—4 times $33\frac{1}{3}$=$$1,33\frac{1}{3}$$, and $$1,33\frac{1}{3}$$ plus $\frac{1}{4}$ of itself=$$1,66\frac{2}{3}$$, hence the $\frac{1}{4}$ that H. is to have in money, is equal to his gain, and this $\frac{1}{4}$ is equal to $\frac{1}{5}$ of the cash price of J.'s pa-

per; consequently $\overline{1,33\frac{1}{3}\times100}\div3=\$41,66\frac{2}{3}$ H. receives, then, $1,33\frac{1}{3}\times100\div,8\frac{1}{3}=1600$ books, J. receives.

8. P and Q barter. P has Irish linen, at 3*s.* 7*d.* per yard in ready money, but in barter 3*s.* 10*d.* Q delivers him cloth at £1, 16*s.* 6*d.* per yard, worth only £1, 13*s.* per yd. Pray, which has the advantage in barter, and how much linen does P give Q for 148 yards of cloth?—36*s.* 6*d.*$\times$148=5402*s.* price of the cloth, then 5402*s.*$\div$3*s.* 10*d.*$=1409\frac{5}{23}$ yards of linen; Q has the advantage, for as 3*s.* 7*d.* : 3*s.*10*d.* : : 33*s.* : £1, 15*s.* $32\frac{17}{43}$*d.* his proportional price.

9. A has 200 yards of linen, at 1*s.* 6*d.* per yd. ready money, which he barters with B at 1*s.* 9*d.* per yd. taking buttons at $7\frac{1}{2}$*d.* per gross, which are worth but 6*d.* How many gross of buttons will pay for the linen, who gets the best bargain, and by how much in the whole, and how much per cent?—200$\times$21*d.* =4200*d.* then 4200$\div7\frac{1}{2}$=560 gross. Ans. Then 200$\times$18*d.* =£15, value of A's linen; then 560$\times$6=£14 value of B's goods. So that B. gained £1 of A, and if 14 : 1 : : 100 : £7, 2*s.* 10*d.* per cent.

EXAMPLES IN COMMISSION AND BROKERAGE.

1. My factor receives $1008 to lay out, after having deducted his commission of 5 per cent. What does his commission amount to?—The just method is as 105 : 5 : : 1008 : $48. Ans.

2. What is the brokerage upon £525, 10*s.* at 5*s.* per cent.?—If £100 : 5*s.* : : £525, 10*s.* : £1, 6*s.* 3+*d.* Ans.

3. If iron worth $4 per cwt. cash is sold for $4,50, on a credit of 8 months, what credit should be allowed on wine worth in cash $224 per pipe, but sold at $242, to make the percentage equal to that on the iron?—If 4 : 4,5 : : 100 : $12\frac{1}{2}$ per cent., then if 100 : 112,5 : : 224 : $252, and 252—224=$28, and 242—224 =$18, then if 28 : $12\frac{1}{2}$: : 18 : $8\frac{1}{28}$ per cent. And if $12\frac{1}{2}$: 8 : : $8\frac{1}{28}$: 5+ months. Ans.

4. Bought 5 hhds. of wine at $1 per gallon, cash; having kept it 3 months and 23 days, I sold it at $1,20 per gallon, on a credit of 5 months; 16 gallons having leaked out while in my possession. What was my cash gain?—63$\times$5=315 gallons in all, which cost $315, then 315—16=299 gallons saved, and 299 $\times\overline{\$1,20-100}$=$59,80 gained on what was saved; then 315$\times$,06=$18, 90 interest on $315 for 1 year, then if 360 : 18,90 : : 263:$13,807,+$16=$29,807, lost by interest and leakage, then $59,80—$29,807=$29,993+gain. Ans.

5. A man bought apples at 5 cents a dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 years; he then *sold all his apples* and pears at a cent apiece, and thus gained 10 *cents. How many* apples did he buy, and how much did they

cost?—1 apple$=\frac{5}{12}$ of a cent, and 8 apples$=\frac{40}{12}$ of a cent, and 5 pears$=\frac{40}{12}$, hence 8 apples and 5 pears$=\frac{80}{12}=\frac{20}{3}=6\frac{2}{3}$ cts.,$=\frac{20}{39}$ of a cent for 1, and 1 sold for $\frac{39}{39}$ of a cent, then $\frac{39}{39}-\frac{20}{39}=\frac{19}{39}$ of a cent gain on 1, then if $\frac{19}{39}:1::19:39$, then $8+5:39::8:24=\frac{1}{3}$ of the apples, therefore 48 apples in all, and if 12 apples : 5 cts. : : 48 : 20 cts. they cost.

EXAMPLES IN INSURANCE.

1. It is required to cover $759, premium 8 per cent. For what sum must the policy be taken?—$\overline{100-8}:100::759:\$825$. Ans.

2. If a policy be taken out for $1250 to cover $500, What is the premium per cent.?—1250 : 500 : : 100 : 40, and 100—40 =$60. Ans.

3. If a policy be taken out for $1250, at 60 per cent, What is the adventure?—100 : $\overline{100-60}$: : 1250 : $500. Ans.

4. A adventured £480,5 from Portland to Charleston, from thence to Jamaica, and from thence home; and the premium was 5 per cent. from port to port. What sum must he take out a policy for, to cover his adventure the voyage round, supposing the risk to be equal out and home, and equal to the several given risks?—First, $100\times100\times100\times480,5=480500000=$ Dividend. Then $\overline{100-5}|^3=857375=$Divisor; then

857375)480500000(£560, 8*s*. 7½*d*. Ans.

5. A adventured £500 from Boston to Baltimore at 3 per cent., from thence to Guadaloupe at 4, from thence to Nantz at 5, and from thence home at 6 per cent. For what sum must he take out a policy to cover his adventure the voyage round, supposing the risk to be equal out and home, and equal to the several given risks?

$$\frac{100\times100\times100\times100\times500}{\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}=£601,278.\ \text{Ans.}$$

6. A adventured £480,5 from Bangor to Richmond, from thence to Jamaica, and from thence home, and the premium was 5 per cent. from port to port. What amount will be equal to the several given premiums, (allowing the risks out and home to be equal,) on the policy which will cover the first adventure of £480,5?—Policy=£560,43125; Then if £560,43125:480,5 : : 100 : £85,7375, and 100—85,7375=£14,2625=amount required.

7. B adventured £480,5 from Newport to Savannah, from thence to Jamaica, and from thence home, to cover which all round he took out a policy for £560, 8*s*. 7½*d*. and the premium

was equal from one place to another. What was the premium per cent.?

$$100-\sqrt{\frac{100\times100\times100\times480,5}{560,43125}}=5 \text{ per cent. Ans.}$$

8. C covers £200 at 6 per cent. from Portsmouth to the W. I. and home again; but the voyage terminating in the W. I. What must the insurer receive per cent.?—$\overline{100-6}:100::200:$ £212,765957 will cover £200 the voyage round, and $100\times100\times200=2000000$, Then

$$\frac{2000000}{212765957}=9400, \text{ and } 100-\sqrt{9400}=3,0465 \text{ per cent. Ans.}$$

EXAMPLES IN DISCOUNT.

1. What is the difference between the interest and discount of $350, for 8 years, at 4 per cent. simple interest?—Interest, of $350, for 8 years at 4 per cent. is $112. And any sum of money, at 6 per cent. simple interest, will double itself in 200 months, and to find the discount of a given sum, for a given time, at a given rate per cent. Add the given time, (in months,) to the number of months required by that sum to double itself at the given rate, for a divisor, to divide the product of the given sum and time, (in months,) and the quotient is the discount. Hence, $\overline{200\times6}\div4=300$ months required for $350, at 4 per cent. to double itself, then $300+\overline{8\times12}(350\times96($84,848 discount of $350, at the given rate and time, then $112-84,848=$27,152. Ans.

2. A merchant has three notes, due to him as follows. One of $300, due in two months; one of $250, due in five months; and one of $180, due three months ago, with interest; the whole of which he now receives. What sum is received on the three notes, allowing money to be worth 6 per cent. a year?—The amount of $180 for 3 months at 6 per cent. is $182,70; and the equated time for 2 mo. and 5 mo., is $3\frac{4}{11}$ months; then $\overline{200+3\frac{4}{11}})\overline{300+250}\times3\frac{4}{11}($9,097 dis., then $\overline{300+250+182,70}-9,097=$723,603. Ans.

3. Sold goods to the amount of $3120, to be paid one half in 3 months, and the other half in 6 months. How much must be discounted for present payment, when money is worth 6 per cent. a year?—The equated time for 3 and 6 months is $4\frac{1}{2}$ months. $\overline{200+4\frac{1}{2}})\overline{3120\times4\frac{1}{2}}($68,655256 discount. Ans.

4. What is the present worth, and discount, of £600 due 3 years hence, at £6 per cent. per annum, compound interest?—Three years time, hence 3d power of

1.06=1.19101)600,000000(£503, 15s. 5¾d. present worth, and £600—£503, 15s. 5¾d.=£96, 4s. 6¼d.=Discount.

CLASS VI,

Consists of questions in Interest, and Annuities.

EXAMPLES IN INTEREST.

1. A had a note against B for $1166,666, dated May 1, 1796, upon which were the following payments, viz.

1. December 25, 1796, $166,666;
2. July 10, 1797, 16,666;
3. September 1, 1798, 50,000;
4. June 14, 1799, 333,333;
5. April 15, 1800, 620,000.

What was due upon it August 3, 1801?

TABLE OF OPERATION.—Set down the sum on which the interest is to be cast, with the time, interest and payments, in columns as follows:

	Principal.	Time.	Interest.	Payments.	Excess.
	Doll's. M.	Mo. D.	Doll's. M.	Doll's. M.	Doll's. M.
1	1166,666 121,167	7 24	45,499	166,666	121,167
2	1045,499	6 15	33,978	16,666	
3	1045,499	13 21	71,616	50,000	
4	1045,499	9 13	49,312	333,333	
	245,093	.	154,906	399,999	245,093
5	800,406 579,847	10 1	40,153	620,000	579,847
	220,559	15 18	17,203		

Then, the last remainder, - - - - $220,559
Interest from the last payment, - - - 17,203

Sum due August 3, 1801, - - - $237,762 Ans.

2. For value received I promise to pay CAROLINE MARTIN, or order, on demand, five hundred dollars, with interest.

O. B. GODING.

Attest, C. F. WATERHOUSE, *Prof. Math.*
Durham, July 1, 1830.

$500

On this note are the following endorsements:

Sept. 16, 1832, - - - -	$120
July 16, 1833, - - - -	200
May 6, 1834, - - - -	150

How much remains due on this note, Jan. 16, 1836?

OPERATION.

Principal and interest to Jan. 16, 1836, (5 yr. 6mo. 15 ds.)= - - - - -	$666,25
Total amount of payments from 1st payment to Jan. 16, 1836,= - - - - - - -	539,25
Remains due Jan. 16, 1836, - - - Ans.	$127,00

3. A bond, dated Jan. 4th, 1837, was given for $1000, int. 6 per cent., and there were payments endorsed upon it as follows:

1st payment, Feb. 19, 1838, - - -	$200
2d payment, June 29, 1839, - - - -	500
3d payment, Nov. 14, 1839, - - -	260

How much remains due on this bond, Dec. 24, 1840?

OPERATION.

Amount, to Feb. 19, 1838, ($13\frac{1}{3}$ mo.) less 1st payment, is balance due Feb. 19, 1838, - - - - -	$867,50
Amount to June 29, 1839, ($16\frac{1}{3}$ mo.) less 2d payment, is balance due, June 29, 1839, - - - -	$438,345
From June 29, to Nov. 14, is less than a year, therefore interest of $438,345, for a year, - -	$26,300
Amount of $438,345 for a year, - - - -	$464,645

Which, less amount of 3d payment for $7\frac{1}{2}$ mo. is balance due June 29, 1840=$194,895, which plus its interest for $5\frac{5}{6}$ mo., is balance due Dec. 24, 1840. - - - Ans. $200,579

4. If $100 in 5 years be allowed to gain $20,50 in what time will any sum of money double itself, at the same rate of interest? —Int. of $100 for 5 years, at 6 per cent. is $30, and if 30 : 6 :: 20,5 : $4\frac{1}{10}$ per cent.; and if 6 : $16\frac{2}{3}$ yrs. : : $4\frac{1}{10}$: $24\frac{6}{41}$ years. Ans.

5. In what time will the interest of $72,60 equal that of $15,25 for 64 days, at any rate of interest?—Int. of $15,25 for 64 days, at 8 per cent. is $0,21399, and the int. of $72,60 for 365 days, at 8 per cent. is $5,808, then if 5,808 : 365 : : ,21399 : $13\frac{5135}{11616}$ days. Ans.

Note.—The three first examples illustrate three different *methods for calculating* interest on partial payments.

EXAMPLES IN ANNUITIES.

1. A man hired a house for 6 years, at $300 a year, but did not pay the rent until the end of the term. How much did the whole amount to, allowing 6 per cent., simple interest?

$\overline{300\times,06\times5 \text{ yrs.}}$ (No. of years less 1) $+600$ (twice the given sum)$=690$, and $\overline{690\times6 \text{ yrs.}}\div2=\$2070$. Ans.

2. If a salary of £120, payable semi annually, remain unpaid 7 years, What would it amount to in that time, at 6 per cent. per annum?—$\frac{1}{2}$ of the ratio (6 per cent.) is 3; and $\frac{1}{2}$ of the annuity, (£120,) is £60, and twice the number of years is 14, the sum of the natural series of which (less 1) is 91; and the int. of £60 for 1 year is 36*s.*; Then $\overline{36\times91}\div20=$£163, 16*s.*= whole int. due, Then £60×14 yrs.=£840, and £840+£163, 16s.=£1003, 16*s.* Ans.

3. If a salary of £120 payable quarterly, remain unpaid 7 years, what would it amount to in that time at 6 per cent. per annum?—$\frac{1}{4}$ of the ratio, (6 per cent.,) is 30*s.* and $\frac{1}{4}$ of the annuity, (£120,) is £30; and 4 times the number of years is 28, the sum of the natural series of which (less 1) is 378; and the int. of £30 for 1 year is 9*s.*; then $\overline{9\times378}\div20=$£170, 2*s.*=whole int. due; Then £30×28 yrs.=£840, and £840+£170, 2*s.*= £1010, 2*s.* Ans.

4. If I hire a house for 3 years, at $250 a year, and wish to make present payment of the whole rent, what sum will be required, allowing discount at 6 per cent.?—Amount of $100 for one year is $106, for two years $112, and for three years $118.

106 : 100 : : 250 : $235,849, present worth for one year,
112 : 100 : : 250 : $223,214, " " " two years,
118 : 100 : : 250 : $211,864, " " " three years.

Present worth, $670,927+ Ans.

5. A man has a pension of $480 a year, and wishes to dispose of it. How much must he receive, allowing 6 per cent.?—If 6 : 100 : : 480 : $8000. Ans.

NOTE.—The five preceding are simple interest examples, and illustrate four different principles, the 2d and 3d, being of a nature, illustrate but one.

6. Suppose the salary of the Vice President of the U. S. A. which is $5000, to remain unpaid 4 years, what would it amount to at 6 per cent. compound interest?

$\overline{1,06\times1,06\times1,06\times1,06-1,}\times5000\div\overline{1,06-1}=\$21873,08.$ Ans.

7. In what time will £60 per annum, payable yearly, amount *to £262,47696,* allowing 6 per cent. compound interest, for the

forbearance of payment?—262,47696×1,06+60=338,2255776, —262,47696=75,7486176,÷60=1,26247696, and 1,26247696÷1,06÷106=1,1236,÷1,06÷1,06=1; the preceding shows 4 divisions by 1,06, therefore 4 years. Ans.

8. A gentleman dying, left a house which rented for $70 a year. The rent was to be put out yearly, at 5 per cent. compound interest, as soon as it was paid in, until his son, who was 14 years old, should become 21 years of age. Required the amount of said annuity for that time?—21—14=7 yearly payments of $70 each, 6 of which draw interest; the first year's rent draws interest for 6 years, the second 5 years, &c. The Table showing the amount of $1 at compound interest for a number of years, shows amount of $70 for 6 years to be $93,806+; and amount of $70 for 5 years=$89,339+; amount of $70 for 4 years=$85,085+; amount of $70 for 3 years=$81,033+; amount of $70 for 2 years=$77,175+; amount of $70 for 1 year=$73,5; and the last $70 does not draw any interest, hence $70=$70; the sum of which amounts plus the $70=$569,938+. Ans.

9. What is due on a pension of $150 a year, payable half-yearly, but forborne 2 years; allowing half-yearly compound interest, at $4\frac{1}{2}$ per cent. per annum?—Per cent. $4\frac{1}{2}\div2$=,0225, which plus unity=amount of one dollar of next to last payment; and 75×1,0225=$76,6875=amount of next to last payment; then 1,0225×1,0225=1,04550625, and, 75×1,04550625 =$78,41296875=amount of second payment; then 1,04550625 ×1,0225=1,069030140625, and 75×1,069030140625=$80,-17726034675=amount of first payment, the sum of which amounts with the last payment, is $310,277729096875. Ans.

10. Suppose a person who has a salary of $600 a year, payable quarterly, to allow it to remain unpaid for 3 years, how much would be due him; allowing quarterly compound interest at 6 per cent. per annum?

Note.—For quarterly payments, take $\frac{1}{4}$ of the ratio, $\frac{1}{4}$ of the annuity, 4 times the number of years; for half yearly, $\frac{1}{2}$ the ratio, $\frac{1}{2}$ the annuity, and double the years.

,06÷4=,015, and $600÷4=$150=each principal of 12 payments, the last of which does not draw interest; then by the rule for finding the other extreme, when either of the extremes, the ratio, and the number of terms in a series of continual proportionals are given, find the amount of the first payment; then by the rule for finding the sum of a series in continual proportionals, when the extremes of that series are given, find the amount due in the preceding question, which is $1955,684+.

11. What is the present worth of the lease of a house for 3 years, the rent being £54, 17s. a year, and the ground rent being 7 guineas a year, the rent payable half-yearly; discount be-

ing allowed at 10 per cent. compound interest?—7 guineas= £7, 7*s*., which taken from £54, 17*s*. leaves £47,5 clear, ½ of which=£23,75, and 3 years doubled=6 years, and ½ of 10 per cent.=5 per cent., and the amount of £1 annuity for 6 years is £6,801913, and amount of £1 for the same time and rate, is £1,340096. Then $\overline{6,801913 \times 23,75} \div 1,340096$=£120, 10*s*. 11¼*d*. Ans.

12. Suppose I would add 5 years to a running lease of 15 years, the rent being $186,375; What ought I to pay down for the favor, discount being allowed at 5 per cent. compound interest?—Amount of $1 annuity for 5 years at 5 per cent. is $5,525631, and the amount of $1 for 20 years at 5 per cent. is $2.653298. Then $\overline{5.525631 \times 186,375} \div 2,653298$=$388,135+ Ans.

13. For how long may an annuity of £60 per annum be purchased for £207,906336762, at 6 per cent. compound interest? $\overline{60l.+£207,906336762-207,906336762 \times 1,06}$=47.525619795 =Divisor. Then 47,525619795)60,000000000(1,26247696, and $\overline{1,26247696 \div 1,06 \div 1,06 \div 1,06 \div 1,06 \div 1,06}$=0, shows 4 divisions by 1,06, hence 4 years. Ans.

14. What is the present worth of 60*l*. payable yearly, for 4 years; but not to commence till two years hence at 6 per cent.? Present worth of the time in being and reversion, (6 years,)= 4,91732. Again by the Table, Pres. worth of the time in being, (2 years,)=1,83339. Then 4,91732—1,83339=3,08393, and 3,08393×60*l*.=£185,0358+. Ans.

15. A person dying, left a piece of land, on a lease, for $250 a year, which, for the first 4 years, he bequeathed to the poor of the parish where he lived; and for the next 6 years it was to belong to the poor of the parish where he was born; and then it was to go into the possession of his niece. Now this young woman, having money, and being desirous to come into the immediate possession of her uncle's property, enters into a compromise with the parishes, on a discount of 6 per cent. What did it cost her?—By the Table, the present worth of $1 annuity, for $\overline{4+6}$ yrs.=7,36008. Then $250×7,36008=$1840,02 cts. Ans.

16. A has a term of 6 years in an estate of 60*l*. per annum, B has a term of 14 years in the same estate, in reversion, after the 6 years are expired; and C has a further term of 16 years, after the expiration of 20 years. I demand the present value of the several terms, at 6 per cent. compound interest? Pres. value of 1*l*. for yrs. $\overline{6+14+16}$=14,61722×60*l*.=877*l*. 0*s*. 7¾*d*.

Of 1*l*. for yrs. $\overline{6+14}$=11,46992× " =688l. 3s. 10¾d

Of 1l. for yrs. 6 - - - - - - 4,91732× " =295l. 0s. 9¼d.

Equal to the present value of A's term.

Then 688.3.10$\frac{3}{4}$—295.0.9$\frac{1}{4}$=393*l.* 3*s.* 1$\frac{1}{2}$*d.*=pres. value of B's.
And 877.0.7$\frac{3}{4}$—688.3.10$\frac{3}{4}$=188*l.* 16*s.* 9*d.*= " " of C's.

17. For a lease of certain profits for 7 years, A offers to pay 300*l.* gratuity, and 300*l.* per annum; B offers 800*l.* gratuity, and 250*l.* per annum; C 1300*l.* gratuity, and 200*l.* per annum; and D bids 2500*l.* without any yearly rent. Which is the best offer, computing interest at 6 per cent. compound interest?—By the Table, Pres. worth of 300*l.* for 7 yrs at 6 per cent. plus 300*l.*= £1974,714=A's offer.

And by the Table, pres. worth of 250*l.* per annum, for 7 yrs. plus 800*l.*=£2195,595=B's offer.

And by the Table, pres. worth of 200*l.* per annum, for 7 yrs. plus 1300*l.*=£2416,476=C's offer.

And £2500=D's offer. The preceding shows that D's offer is best.

18. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for £185,035876, at 6 per cent.?

Thus, $\frac{185,036\times1,26247\times1,26247\times\overline{1,06-1}}{\overline{1,26247-1}}=60l.$ Ans.

19. What is the value of a rent of 60*l.* per annum, to commence 2 years hence, allowing the purchaser 6 per cent.?—Thus, ,06)60,00)1000*l.*=its present worth. Then $\overline{1,06\times1,06}$ =1,1236)10000000(889*l.* 19*s.* 11*d.* present worth required. Ans.

And if the Contra. of the preceding question is required. Thus, price of the reversion $\frac{889,9966\times\overline{1,06}|^2\times,06}{}=60l.$ Ans.

20. Which is the most advantageous, a term of 15 years in an estate of 100*l.* yearly, or the reversion of said estate forever after the said 15 years, computing interest at 5 per cent. per annum, compound interest?—Present worth of 1*l.* ann. for 15 yrs. =10,379658,×100*l.*=£1037,9658=pres. worth of 100*l.* for 15 years; and 100*l.*÷,05=2000*l.* value of the estate. Amount of 1*l.* for 15 years at 5 per cent. is £2,078928, and 2000÷2,078928 =£962,0342=present worth of the estate in reversion, then £1037,9658—£962,0342=£75,9316 that the present worth of the rent is worth, more, than the present worth of the whole estate.

NOTE.—The preceding examples illustrate several principles.

IMPORTANT PROBLEMS.

I. To find any number of mean proportionals between two given numbers.

RULE.—Divide the greater number by the less, and the quotient will be that power of the ratio which is denoted by the number of means plus 1, and the corresponding root of this quotient will be the ratio. Then the product of the ratio and the less number will be one of the means; the product of this mean and the ratio will be another mean; and so on till the required means are found.

EXAMPLES.

1. What is the mean proportional between 24 and 96?—$96 \div 24 = 4$, and $\sqrt[2]{4} = 2 =$ ratio; then $24 \times 2 = 48$. Ans.

2. What are the two mean proportionals between 6 and 750? $750 \div 6 = 125$, and $\sqrt[3]{125} = 5$. Then $6 \times 5 = 30 =$ least mean, and $30 \times 5 = 150 =$ greatest.

3. Find three mean proportionals between 5 and 1280.—$1280 \div 5 = 256$, and $\sqrt[4]{256} = 4$. Then $5 \times 4 = 20 =$ least, and $20 \times 4 = 80 =$ next, and $80 \times 4 = 320 =$ last.

II. To find the denominator to the remainder that sometimes occurs when finding the square or cube root of a given number.

RULE.—Double the root is the Den. to a Rem. in square root, and triple the square of the root plus triple the root is the Den. to a Remainder in finding the cube root of a given number.

EXAMPLES.

1. In finding the square root of a certain number, the root is 12 and the remainder 15. What fraction belongs to the root?—Thus, $12 \times 2 = 24 =$ Den. to 15, hence $\frac{15}{24}$. Ans.

2. In finding the cube root of a certain number, the root is 12 and the remainder 15. What fraction belongs to said root?—$\overline{12}|^2 \times 3 + \overline{12 \times 3} = 468 =$ Den.; or $\overline{12 \times 12 \times 3} + \overline{12 \times 3} = 468$, then $\frac{15}{468}$. Ans.

III.—The sum of any two numbers, and their product being given to find each number.

RULE.—From the square of their sum, subtract four times their product, and extract the square root of the remainder, which will be the difference of the two numbers. You will then have their sum and difference to find the numbers by Prob. I. See PROBLEMS.

EXAMPLES.

1. Bought a certain quantity of cloth for $573,75, and if the *number of cents* which it cost per yard, was added to the number

of yards bought, the sum would be 480. How many yards did I buy, and at what price per yard?—$\overline{480}|^2$—573,75×4=900, and $\sqrt[2]{900}$=30=Diff.; $\overline{480—30}$÷2=225 cents per yard, and 480—225=255 yds.

2. If I lay out a lot of land in an oblong form, containing 7 acres, 1 rood, and 10 rods, and taking just 142 rods of wall to enclose it. How many rods long, and how many wide is said lot?—7 acres, 1 rood, and 10 rods=1170 rods, and 142 rods is twice its length and width, then 142÷2=71, and 71×71=5041=square of the sum, then rods 1170×4=4680=4 times the product, and 5041—4680=361, and $\sqrt[2]{361}$=19=Diff. of the numbers, then $\overline{71—19}$÷2=26 rods wide, and 71—26=45 rods long.

IV. The difference of two numbers, and their product, given to find the numbers.

Rule.—Square the difference of the two numbers, then add one fourth of the square to the product, and twice the square root of this sum is the sum of the two numbers. You will then have their sum and difference to find the numbers by Prob. I. See Problems.

EXAMPLE.

A man had a field in the form of a parallelogram, the lengt of which was 40 rods more than the breadth, and the area 3825 rods. Required the length and breadth.—40×40=1600, and 1600÷4=400, which plus 3825=4225, and $\sqrt[2]{4225}$=65, and 65×2=130=the sum of the numbers, then $\overline{130—40}$÷2=45 rods in breadth and 130—45=85 rods in length.

V. The sum of two numbers, and the sum of their cubes given to find the numbers.

Rule.—From the cube of the sum, subtract the sum of the cubes, and divide the remainder by three times the sum of the numbers, and the quotient will be the product of the two numbers. Then, having the sum and product given, the numbers may be found by Prob. III. See Important Problems.

EXAMPLE.

A farmer sold two cubical stacks of hay. A side of the greater stack, added to a side of the less, was 20 feet, and both stacks contain 2240 cubic feet. Required the length of a side of each. 20×20×20=8000=cube of the sum, and $\overline{8000—2240}$÷$\overline{20}$×3=96=product of the numbers, then $\overline{20×20—96}$×4=16, and $\sqrt[2]{16}$=4=Diff. then $\overline{20—4}$÷2=8 ft. side of the smaller stack, *and 20—8=12 ft.* side of the larger stack.

VI. The sum and difference of the squares of two numbers given, to find these numbers.

RULE.—From the sum take the difference, and half the remainder is the square of the less, which, taken from the sum of the squares, will give the square of the greater, the square root of each of which is each number.

EXAMPLE.

A and B have between them a number of apples, which are to be so divided, that the sum of their squares may be 208, and the difference of their squares 80. If A has the most, how many has each?—$\overline{208-80}\div2=64$, and $\sqrt[2]{64}=8=$B's number; then $\sqrt[2]{\overline{208-64}}=12=$A's number.

VII. The sum of two numbers, and the sum of their squares given to find those numbers.

RULE.—From the square of their sum take the sum of their squares; then from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers. You will then proceed by Prob. I. See PROBLEMS.

EXAMPLE.

A and B have each a square lot, of which A's is the larger. The length of one side of A's plus the length of one side of B's, is 110 rods, and the quantity of land contained in both, is 6100 rods. Required the length of a side of each, and the area. $\overline{110\times110}-6100=6000$, and $6100-6000=100$, and $\sqrt[2]{100}=10=$Diff.; $\overline{110-10}\div2=50=$side of B's; and $110-50=60=$side of A's. Then $50\times50=2500$ rods=area of B's, and $60\times60=3600$, or $6100-2500=3600$ rods=area of A's.

VIII. The difference of two numbers, and the sum of their squares given, to find those numbers.

RULE.—From the sum of their squares take the square of their difference. To the sum of their squares add the remainder, and the square root of this sum will be the sum of the required numbers; then with the sum and difference proceed by Prob. I. See PROBLEMS.

EXAMPLE.

A and B have a number of guineas, and A is to have 50 more than B, and the sum of the squares of their respective shares is to be 12500. What number is each to have?—$12500-\overline{50\times50}=10000$, and $\sqrt[2]{\overline{12500+10000}}=150=$sum of their shares. Then $150-50=100=$A's; and $150-100=50=$B's.

IX. The sum of the squares of two numbers, and the square of their half sum given, to find those numbers.

RULE.—From the sum of the squares take twice the square of

the half sum, and double the square root of half the remainder will be the difference, and the square root of the sum of the squares will be the sum of the numbers. Then with the sum and difference proceed as before directed.

EXAMPLE.

The sum of the squares of two numbers is 3161, and the square of their half sum is 1560,25. Required those numbers.—3161—$\overline{1560,25 \times 2}$=40,5, and $\sqrt[2]{40,5 \div 2 \times 2}$=9=Diff. of the two numbers. Then, $\sqrt[2]{1560,25}$=39,5, and 39,5×2=79=sum; then 79—9÷2=35=less number, and 79—35=44=greater.

X. To find the true weight of any commodity when weighed in each scale of a balance the beam of which is unequally divided.

Rule.—The square root of the product of the different weights is the true weight.

EXAMPLES.

1. Suppose the weight of a bar of silver, in one scale, to be 10 *oz.*, and in the other scale 12 *oz.* Required the true weight of that body.—Thus, $\sqrt[2]{12 \times 10}$=10,954 ounces. Ans.

2. A cheese in one scale weighed 76 lbs., in the other 56 lbs. What was its true weight?—$\sqrt[2]{76 \times 56}$=65 lbs. and 31 remainder, then 65×2=130=Den. hence $65\frac{31}{130}$ lbs. Ans.

XI. To find the force, velocity, weight, or capacity of the other of two bodies, when two properties of one, and one property of the other is given.

Rule.—Proceed by Proportion.

Note.—The effects of degrees of light, heat, attraction, and capacities of water pipes are in proportion to their squares; and the gravity of bodies, from the earth's centre, decrease as the square of their distances, in semi-diameters of the earth; and the capacity of solids are in proportion to the cubes of their properties.

EXAMPLES.

1. Suppose the battering ram of Vespasian weighed 60000 lbs; that it was moved at the rate of 24 feet in a second, and that this was sufficient to demolish the walls of Jerusalem. With what velocity must a cannon ball which weighs 42 lbs. be moved, to do the same execution?—Thus, if 60000 : 24 : : 42 : $342857\frac{1}{7}$ feet in a second. Ans.

2. A body weighing 30 lbs. is impelled by such a force as to send it 20 rods in a second. With what velocity would a body weighing 12 lbs. move, if it were impelled by the same force?—Thus, if 12 : 20 : : 30 : 50 rods. Ans.

3. *Admit 10 hhds.* of water are discharged through a leaden *pipe of* $2\frac{1}{2}$ *inches* diameter, in a certain time; What must be

the diameter of another pipe, that shall discharge four times as much in the same time?—4=given proportion, $2\frac{1}{2}$=2,5; then 2,5×2,5=6,25=square; and 6,25×4=25, and $\sqrt[2]{25}$=5 inc. diameter. Ans.

4. If the diameter of a circle be 12 in. what is the diameter of one $\frac{1}{4}$ as large?—Thus 12×12=144, and 144÷4=36, and $\sqrt[2]{36}$=6 inches. Ans.

5. If the earth's mean distance from the sun be 95 millions of miles, at what distance from him must another body be placed, that it may receive a degree of light and heat double to that of the earth? Thus, $\sqrt[2]{\frac{95000000}{2}}$=67175144+ ms. Ans.

6. A mirror of 47 inches in diameter, and 38 inches in focal distance, will condense the solar rays to 17,257 times a degree of heat; and the power of these glasses for burning are as the area of the lens directly is to the square of the focus distance inversely. Hence the following:

If a mirror that has 47 inch diameter, and 38 inch focus, condense the solar rays to 17,257 times a degree of heat, and melts copper ore in 8 seconds, how many times a degree of heat, and in what time could copper ore be melted by a mirror that has 36 inch diameter and 80 inch focus?

First, $\frac{47\times47\times11\div14 : 38\times38}{36\times36\times11\div14 : 80\times80}$ :: 17,257 : 44,873+times. Ans.

Then, if 17,257 : 8 :: 44,873 : 3,07+ sec. Ans.

7. A's magnet is 3 ft. and B's 6 ft. from a bed of attracting substance. What power of attraction does each magnet realize?—Say (inversely,) as $\overline{6\times6}$: 1 :: $\overline{3\times3}$: 4, hence A's magnet is attracted four fold the most.

8. If a body weighs 400 lbs. at 2000 miles above the earth's surface, what would it weigh at the surface, if the earth's semi-diameter is 4000 miles?—From the centre to the given height, is $1\frac{1}{2}$ semi diameter, then $\overline{1,5\times1,5}\times400$=900 lbs. Ans.

9. If the reverse of the preceding is required, Thus, 900÷$\overline{1,5\times1,5}$=400 lbs. Ans.

10. A certain body on the surface of the earth, weighs 180 lbs; How high must it be carried to weigh but 20 lbs.?

Thus, $\sqrt[2]{180\div20}$=3. Ans. 3 semi-diameters, which is 8000 miles above its surface.

12. How high must a ball be raised to lose half its weight?—As 1 : 4000×4000 :: 2 : 32000000, and $\sqrt[2]{32000000}$=5656,85, which less 4000=1656,85 ms. Ans.

13. If the attraction of the moon raise a tide on the earth five feet high, and the attraction of one of those bodies to the other's surface is directly as its quantity of matter, and inversely as its

diameter; What will be the height of a tide, raised by the earth upon the moon's surface, under similar circumstances?—As $\overline{2182}|^3 \times 494 : 5 :: \overline{8000}|^3 \times 400 : 199,3$; then as $2182 : 199,3 :: 8000 : 54,8$ feet. Ans.

14. I. If the diameters of two globes be equal, and their densities different, the weight of a body on their surfaces will be as their densities.

II. If their densities be equal, and their diameters different, the weight of a body will be as $\frac{1}{3}$ of their circumferences.

III. If their diameters and densities be both different, the weight will be as $\frac{2}{3}$ of their semi-diameters multiplied by their densities.

If a stone weigh 100 lbs. at the surface of the earth, what will it weigh at the surfaces of the sun and the several planets, if their densities and diameters are as in the following Table.

	Sun.	Jupiter.	Saturn.	Earth.	Moon.
Their Densities,	100	94,5	67	400	494,
Diam. in miles,	776970	135079	98566	6875	1869,35
Semi-diameter,	388485	67539,5	49283	3437,5	934,67
$\frac{2}{3}$ Semi-diam.	258990	44693	32855,33	2295	623,11

Then, as $2295 \times 400 : 100 ::$ $\left\{ \begin{array}{l} 258990 \times 100 : 2821,24 \text{ lb. at the sun.} \\ 44693 \times 94,5 : 460 \text{ lb. at Jupiter.} \\ 32855,33 \times 67 : 239,8 \text{ lb. at Saturn.} \\ 623,11 \times 494 : 33,5 \text{ lb at the moon.} \end{array} \right\}$ Ans.

15. There are two planets of equal density; the diameter of the less is to that of the larger as 2 to 9. What is the ratio of their solidities?—$2 \times 2 \times 2 = 8$; $9 \times 9 \times 9 = 729$, hence, $\frac{8}{729}$; or, as 8 to 729. Ans.

16. If a globe 6 inches in diameter weighs 25 lbs. What is the weight of another of the like metal whose diameter is 3 in.?

$6 \times 6 \times 6 = 216$ the cube of the given diameter; $3 \times 3 \times 3 = 27$ the cube of the required diameter. As $216 : 25 :: 27 : 3,12+$ lbs. Ans.

17. If a cube of silver, whose side is 2 inches, be worth $20, What should the side of a cube of like silver be, whose value would be 8 times as much?—$2 \times 2 \times 2 = 8$, and $8 \times 8 = 64$. Then, $\sqrt[3]{64} = 4$ inches. Ans.

18. A cooper having a cask 40 inches long and 32 inches at the bung diameter, is ordered to make another cask of the same shape, but which shall hold just twice as much. What will be the bung diameter and length of the new cask?—$40 \times 40 \times 40 \times 2 = 128000$; then $\sqrt[3]{128000} = 50,3+$ inc. length. Ans. $32 \times 32 \times 32 \times 2 = 65536$; and $\sqrt[3]{65536} = 40,3+$ inc. bung diam. Ans.

19. If the diameter of the sun is 112 times of that of the *earth, how many globes* like the earth are equal in size to the sun?

$\overline{8000}|^3 \times 112 \div \overline{8000}|^3 = 1404928$ globes. Ans.

20. there is a box that is 4 feet wide, 4 feet high, and 4 feet long. What is the side of another box that shall contain $\frac{1}{8}$ of the quantity?—$\overline{4\times4\times4}\div8=8$, and $\sqrt[3]{8}=2$ feet. Ans.

21. If a cable 12 inches about, require an anchor of 18 cwt., of what weight must an anchor be for a 15 inch cable?—As $12\times12\times12 : 18$ cwt. : : $15\times15\times15 : 35,15625$ cwt.

If the reverse is asked, thus, $35,15625 : \overline{15}|^3 :: 18 : 1728$, and $\sqrt[3]{1728}=12$ in. Ans.

22. If a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold, What are the dimensions of a ship of 200 tons, of the same mould and shape?—$\overline{89}|^3=704969=$ cubed keel, then if $500 : 704969 :: 200 : 281987,6$ and $\sqrt[3]{281987.6}=65,57$ feet=required keel. As $89 : 65,57 :: 36 : 26\frac{1}{4}$ ft. beam nearly. As $89 : 65,57 :: 16 : 11,7$ ft. depth of the hold.

XII. The velocity (OF A FALLING BODY) given, to find the space fallen through.

RULE.—Divide the velocity by 8, and the square of the quotient will be the distance fallen through to acquire that velocity.

EXAMPLE.

If the velocity of a cannon ball is $\frac{1}{8}$ of a mile per sec., From what height must a body fall to acquire the same velocity?—$\frac{1}{8}$ of a mile=660 ft., and $660\div8=82,5$, and $82,5\times82,5=6806\frac{1}{4}$ ft.$=1\frac{37}{128}$ mile. Ans.

XIII. The time given, to find the space fallen through.

RULE.—Multiply the time by 4, and the square of the product will be the space fallen through in the given time.

EXAMPLES.

1. What is the difference between the depth of two wells, into each of which should a stone be dropped at the same instant, one would reach the bottom in 5 seconds, and the other in 3?—$5\times4=20$, and $20\times20=400$; then $3\times4=12$, and $12\times12=144$; then $400-144=256$ feet. Ans.

2. A ball was seen to fall half the way from the top of a tower in the last second of time. How long was it in descending, and what was its height before its descent?—The $\sqrt[2]{1}=1$, and $\sqrt[2]{2}=1,4142$; and $1,4142-1=,4142$; then as $,4142 : 1,4142 :: 1 : 3,414+$ seconds=time of its descent, then $\overline{3,414\times4}|^2=186,424+$ feet. Ans.

NOTE.—Falling bodies fall 16 feet the first second, and increase fourfold in velocity every second; and ascending bodies are retarded in the same ratio that descending bodies are accellerated; therefore, if a ball, discharged from a gun, returned to the earth in *12 seconds*, How high did it ascend?—Thus, the ball being

half of the time, or 6 seconds, in its ascent, hence, $6\times4=24$, and $24\times24=576$ feet. Ans.

XIV. The velocity per second given, to find the time.

RULE.—Divide the given velocity by 8, and $\frac{1}{4}$ of the quotient is the time.

EXAMPLE.

How long must a bullet be falling to acquire the velocity of 160 feet per second?—Thus, $160\div8=20$, and $20\div4=5$ seconds. Ans.

NOTE.—In finding the space, velocity, time, or force, that descending or ascending bodies are accelerated or retarded, feet, seconds, and pounds are the denominations used and obtained.

XV. Given, the space fallen through, to find the time it has been falling.

RULE.—Divide the square root of the space fallen through by 4, and the quotient will be the time in which it was falling.

EXAMPLE.

In what time will a musket ball, dropped from the top of a steeple 484 feet high, come to the ground?—$\sqrt[2]{484}=22$, and $22\div4=5\frac{1}{2}$ seconds. Ans.

XVI. To find the velocity per second, with which a heavy body will begin to descend, at any distance from the earth's surface.

RULE,—As the square of the earth's semi-diameter is to 16 feet, so is the square of any other distance from the earth's centre, inversely, to the velocity with which it begins to descend per second.

EXAMPLE.

With what velocity per second will a ball begin to descend, if raised 3000 miles above the earth's surface?—As $4000\times4000 : 16 :: \overline{4000+3000}\times\overline{4000+3000} : 5{,}22449$ feet. Ans. And if the height is required, and the velocity given, thus, as $16 : 4000\times4000 :: 5{,}22449 : 49000000$, and $\sqrt[2]{49000000}-4000=3000$ miles. Ans.

XVII. To find the mean velocity of a falling body.

RULE.—Divide the space fallen through by the number of seconds it was falling, and the quotient is the mean velocity.

EXAMPLE.

A ball dropped from the top of a steeple 484 feet high will reach the earth in $5\frac{1}{2}$ seconds. Required its mean velocity?—$484\div5{,}5=88$ ft. per second. Ans.

XVIII. To find the velocity per second that a falling body *has acquired at the* end of any given period of time.

RULE.—Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity.

NOTE.—If a question requires the reverse of the problem, reverse the rule.

EXAMPLE.

There is a flume, one end of which is $2\frac{1}{2}$ feet higher than the other. What would be the velocity of a stream of water through it per second?—$2,5\times64=160$, and $\sqrt[2]{160}=12,649$ feet. Ans.

Required the reverse. $\overline{12,649\times12,649}\div64=2\frac{1}{2}$ feet. Ans.

XIX. The weight of a body, and the space fallen through, given, to find the force with which it will strike.

RULE.—Find the velocity by Prob. XVIII, and multiply it by the weight, the product is the force required in lbs. But if the weight, and striking force, is given, to find the space fallen through;

RULE.—Divide the force by the weight, the quotient is the velocity. Then divide the square of the velocity by 64, and the quotient is the space fallen through, in feet.

EXAMPLES.

1. If the rammer used in driving the piles of Charlestown Bridge, weighed $2\frac{1}{4}$ tons, and fell through a space of 10 feet, with what force did it strike the pile?—$2\frac{1}{4}$ tons$=4500$ lbs. $\sqrt[2]{10\times64}=25,3=$velocity, and $25,3\times4500=113850$ lbs. momentum. Ans. The reverse; $113850\div4500=25,3$, and $\overline{25,3\times25,3}\div64=10$ feet. Ans.

NOTE.—To find the force that a fluid, moving with a given velocity, strikes upon a fixed obstacle: By Prob. XVIII, find the fall that will produce the given velocity; multiply said fall by 62,5 lbs. avoirdupois, for clean river water, by 63 lbs. for dirty water, and by 64 for sea water. Therefore, What is the pressure against an obstacle 15 feet wide and 4 feet high, by a stream of clear water moving against it at the rate of 5 feet per second?—$\overline{5\times5}\div64=\frac{25}{64}=,39$ of a foot fall, then $62,5\times,39=24,375$ lbs. against a square foot; then $15\times4\times24,375=1462,5$ lbs. velocity; then $1462,5\times5=$motion$=7312,5$ lbs. Ans.

2. A miller has a head of water 4 feet above the sluice; How high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time?—$\sqrt[2]{4}=2$, and half as much again as 2, is $2+1=3$, then $3\times3=9$ feet. Ans.

XX. To find the quantity of pressure against the sluice or bank, which pens a body of water.

RULE.—Multiply the area of the sluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of water,) in feet, and that product again by $62\frac{1}{2}$ for clear, or

64,4 for sea water, and the product is the number of pounds required.

EXAMPLE.

If the length of a sluice be 30 feet, and the depth of the water 4 feet, What is the pressure against the side of the sluice?—$30\times4=120$ feet, the area of the side, and 120×2 (the depth of the centre of gravity,) gives 240 cubic feet, and $240\times62,5=15000$ lbs. Ans.

XXI. To find the perpendicular pressure of fluids on the bottom of vessels.

RULE.—Multiply the area of the bottom by the altitude of the fluid, and that product by its specific gravity, gives the pressure required.

EXAMPLE.

Suppose a vessel 3 feet wide, 5 feet long, and 4 feet high; What is the pressure on the bottom, it being filled with water to the brim?—$3\times5=15$ square feet, the area of the bottom, and $15\times4=60$ cubic feet, and $60\times62,5=3750$ lbs.=1 ton, 13 cwt. 1 qr, 26 lbs. Ans.

XXII. To find the length of a pendulum, that will swing any given time.

RULE.—Multiply the square of the seconds in any given time by 39,2, and the product is the length required, in inches.

EXAMPLE.

Required the lengths of several pendulums, which will respectively swing $\frac{1}{4}$ seconds, $\frac{1}{2}$ seconds, seconds, minutes, and hours?—$,25\times,25\times39,2=2,45$ inches for $\frac{1}{4}$ sec; $,5\times,5\times39,2=9,8$ in. for $\frac{1}{2}$ sec.; $1\times1\times39,2=39,2$ in for sec.; $60\times60\times39,2=141120$ in.=11760 feet=2 miles and 1200 feet for minutes; and 1 hour=3600 sec., then $3600\times3600\times39,2=$the inches in 8018 miles and 96 feet for hours.

XXIII. To find the time which a pendulum of any given length will swing.

RULE.—Divide the given length by 39,2, and the quotient will be the square of the time in seconds.

EXAMPLE.

How often will a pendulum of 9,8 inches vibrate in a second? Thus, $9,8\div39,2=,25$, and $\sqrt[2]{,25}=,5$ of a second; i. e. it vibrates half seconds. Ans.

XXIV. To find the depth of a well by dropping a stone into it, also the time of the stone's descent and of the sound's ascent.

RULE.—1. Take a line of any length, and by the last Prob. *find the time from the* dropping of the stone until you hear it

strike the bottom. 2. Multiply 73088 by the number of seconds till you hear the stone strike the bottom. 3. To this product add 1304164, and from the square root of the sum take 1142. 4. Divide the square of the remainder by 64, and the quotient will be the depth of the well in feet. 5. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which, being taken from the whole time, will leave the time of the stone's descent in seconds.

EXAMPLE.

Suppose I drop a stone into a well, and a string with a plummet, which measured to the middle of a ball 25 inches, made 5 vibrations before I heard the stone strike the bottom; Required the depth, time of the stone's descent, and of the sound's ascent? $\sqrt[2]{25 \div 39,2} = ,7985$, and $,7985 \times 5 = 4$ seconds to the hearing of it strike; then $\sqrt[2]{,73088 \times 4 + 1304164} - 1142 = 121,53$, and $121,53 \times 121,53 \div 64 = 230,77$ feet, the depth; and $230,77 \div 1142 = ,2$ of a second, the time of the sound's ascent; and $4 - ,2 = 3,8$ seconds, the time of the stone's descent.

XXV. Given, the weight to be raised by a balloon, to find its diameter.

RULE.—As the specific difference between common and inflammable air, is to one cubic foot, so is any weight to be raised to the cubic feet contained in the balloon. Divide the cubic feet by ,5236, and the cube root of the quotient will be the diameter required; but to raise it, the diameter must be something greater, or the weight something less.

EXAMPLE.

I would construct a spherical balloon of sufficient capacity to ascend with 4 persons weighing one with another 160 lbs., and the balloon and a bag of sand weighing 60 lbs. Required the diameter of the balloon. By the Table of Specific Gravities, I find a cubic foot of common air weighs 1,25 oz., and a cubic foot of inflammable air ,12 oz. both avoirdupois; then $1,25 - ,12 = 1,13$ oz. difference. And $160 \times 4 + 60 = 700$ lbs. $= 11200$ oz.; then $1,13 : 1 :: 11200 : 9911,5044$ cubic feet. And $\sqrt[3]{9911,5044} \div ,5236 = 26,65$ feet, the required diameter. Ans.

XXVI. Given, the diameter of a balloon, to find the weight it is capable of raising.

RULE.—Multiply the cube of the diameter by ,5236, and the product will be the content in cubic feet. Then, as 1 cubic foot is to the specific difference between common and inflammable air, so is the content of the balloon to the weight it will raise.

EXAMPLE.

The diameter of a balloon is 26,65 feet. What weight is it capable of raising?—26,65×26,65×26,65×,5236=9911,4+ cubic feet. And as 1 : 1,13 : : 9911,4+ : the ounces in 700 lbs., nearly. Ans.

XXVII. To find the magnitude of any thing, when the weight is known.

RULE.—Divide the weight by the specific gravity in the Table, and the quotient will be the magnitude sought.

EXAMPLE.

What is the magnitude of several fragments of clear glass, whose weight is 13 oz.?—13÷2600=,005 of a cubic foot, and ,005×1728=8,640 cubic inches. Ans.

XXVIII. The bulk and weight of any body given, to find its specific gravity.

RULE.—Divide the weight by the bulk, and the quotient is the specific gravity.

EXAMPLE.

Suppose a piece of marble contains 8 cubic feet, and weighs 1353½ lbs., or 21656 ounces. What is its specific gravity?—21656÷8×2707, the specific gravity as required by the Table.

NOTE.—If the bulk of a body be multiplied by its specific gravity, the product will be its absolute weight. What weight of lead will cover a house, the area of its roof being 6000 feet, and the thickness of the lead $\frac{1}{120}$ of a foot?—6000×$\frac{1}{120}$=50 cubic feet, and its specific gravity 11325×50=566250 oz.= 15 tons, 15 cwt. 3 qrs. 26 lbs. 10 oz. Ans.

XXIX. Given, the power, the prop or fulcrum's place, and the length of the lever, to find the weight that may be raised.

RULE.—As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop, or fulcrum, and the point where the power is applied, so is the power to the weight which it will balance.

NOTE.—By a due consideration this rule may be varied to suit any question that ranks with this mechanical power.

EXAMPLES.

1. If a man, weighing 160 lbs., rest on the end of a lever 10 feet long, What weight will he balance on the other end, supposing the prop one foot from the weight?—The distance between the weight and prop being 1 foot, the distance from the prop to the power is 10—1=9 feet, hence as 1 : 9 : : 160 : 1440 lbs. Ans.

NOTE 1.—Reverse the work to find the power that will raise a *given weight. Thus,* as 9 : 1 : : 1440 : 160 lbs. power. Ans.

NOTE 2.—If to find the distance between the prop and power that will balance a given weight. Thus, as 160 : 1440 : : 1 : 9 feet. Ans.

NOTE 3.—If to find the distance between the prop and weight to be balanced. Thus, as 1440 : 160 : : 9 : 1 foot. Ans.

2. In giving directions for making a chaise, the length of the shafts, between the axletree and back-band, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. A advised to place it 30 inches, B 20 inches before the axletree. Now supposing the body of the chaise and its burthen are 420 lbs. What will the beast in both cases bear, more than his harness?

9 ft.=108 inches. Then, as 108 : 420 : : $\left\{\begin{array}{l} 30 : 116\frac{2}{3} \text{ lbs.} \\ 20 : 77\frac{7}{9} \text{ lbs.} \end{array}\right\}$ Ans.

XXX. Given, three parts of the wheel and axle, to find the fourth.

RULE.—As the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel, to the weight suspended by the axle.

NOTE.—By little consideration, this rule may be varied to suit any question that ranks with this mechanical power.

EXAMPLE.

A would make a windlass in such a manner that one pound applied to the wheel, should be equal to ten pounds suspended from the axle; now, supposing the axle 6 inches in diameter, required the diameter of the wheel?

Inversely, as 10 : 6 : : 1 : 60 inches. Ans.

NOTE 1.—If from the preceding 3 parts be taken, to vary the rule and to find the axle's diameter.

Thus, inversely, as 1 : 60 : : 10 : 6 inches. Ans.

NOTE 2.—Again 3 parts taken, to find the power to be applied to the wheel. Inversely, as 6 : 10 : : 60 : 1 lb. Ans.

NOTE 3.—If to find what weight at the axle will balance a given power. Inversely, as 60 : 1 : : 6 : 10 lbs. Ans.

XXXI.—The number of movable pulleys and the power given, to find what weight may be raised.

RULE.—As 1 is to twice the number of movable pulleys, so is the power to the weight.

NOTE.—Reverse the rule to find the power.

EXAMPLES.

1. What weight would balance a power of 45 lbs. that is applied to a cord that runs over 3 movable pulleys?

Thus, $1 : 3 \times 2 : : 45 : 270$ lbs. Ans.

The reverse, thus, $270 : \overline{3 \times 2} : : 1 : 45$ lbs. Ans.

2. If a cord, which runs over 3 movable pulleys, be attached to an axle 4 inches in diameter, the wheel of the axle being 38 inches in diameter, and a power of 20 pounds be exerted at the circumference of the wheel, What weight would be raised under the pulleys?—Thus, $4 : 38 :: \overline{20 \times 3 \times 2} : 1440$ lbs. Ans.

XXXII. The nature of the Inclined Plane is illustrated by the following EXAMPLES, each of which illustrates a distinct principle.

1. A certain inclined plane is 16 feet in length, and 7 feet in perpendicular height. What weight might be drawn up this plane by a power, which, if exerted on a cord over a single fixed pulley, would raise 25 lbs.?

As the height 7 : 16 length : : 25 power : $57\frac{1}{7}$ lbs. weight. Ans.

2. The reverse is, as the length 16 : 7 height : : $57\frac{1}{7}$ weight : 25 lbs. power. Ans.

3. If to find the length of the plane, when the height and power to be applied is given, say, As the power 25 : 7 height : : $57\frac{1}{7}$: 16 feet=length. Ans.

4. If to find the height, if the other three parts are given, thus $57\frac{1}{7}$: 16 : : 25 : 7 feet. Ans.

NOTE.—The wedge is of the same nature as the inclined plane, calling the breadth of the head of the wedge the perpendicular height of the plane, and the length of its side the length of the plane, and the force acting against the head of the wedge, the power.

XXXIII. The mechanical power of the screw has the following

RULE.—Find the circumference of the circle described by the end of the lever; then, as that circumference is to the distance between the spiral threads of the screw, so is the weight to be raised to the power that will raise it, abating the friction, which is at a medium, of $\frac{1}{3}$ part of the effect of the machine.

NOTE.—The following example is varied to show the distinct principles of the preceding rule.

EXAMPLE.

There is a screw whose threads are an inch asunder, the lever by which it is turned 30 inches long, and the weight to be raised 2240 lbs. What power or force must be applied to the end of the lever to raise the weight?—The lever being the semidiameter of the circle, the diameter is 60 inches. Then $3,1416 \times 60 = 188,496$ inches, the circumference; therefore, as 188,496 : 1 : : 2240 : 11,88. Ans.

1. If the requisite parts of the preceding example be taken to *find the weight* to be raised, say, As 1 : 188,496 : : 11,88 : 2240 *lbs. Ans.*

2. And if the distance between the threads be required, proceed thus, As 2240 : 11,88 : : 188,496 : 1 inch, nearly. Ans.

3. Again, to find the length of the lever, if the requisite parts are given; as 11,88 : 2240 : : 1 : 188,5. Then, as 355 : 113 : : 188,5 : 60, and 60÷2=30 inches. Ans.

NOTE.—Inches and pounds are the denominations used and obtained in the calculation of mechanical power.

XXXIV. To get a right line from an object in any direction.

RULE.—Keep set up, forward and back of one as they proceed, any convenient number of stakes, in range with the instrument and with one another.

XXXV. To find a right line between two objects, (whether one of the objects can or cannot be seen from the other,) when the distance between them is given.

RULE.—Proceed on a right line, for the given distance, from one object toward the other, as nigh as the judgment directs. Seek the other object, and find how far you come out to the right or left of it, then say, As the given distance is to the distance that you come out to the right or left of the other object, so is any part of the given distance, to a distance that should be measured from the taken right line and parallel to the distance that you come out to the right or left of the other object, at the end of which measured distance stick up a stake; and thus, get as many stakes as desirable between the objects, which range of stakes shows the right line between the two given objects. Or, take a bearing and proceed as before, and find the distance that you come out to the right or left of the other object, then find the circumference to what twice the given distance is the diameeter of and then say, As this circumference is to 360 degrees, so is the distance that you come out to the right or left of the other object, to the Degrees, &c. that should be added to, or subtracted from, (according as the case requires,) the reverse of the taken bearing, to produce the bearing that you will proceed by, from the other object to the object you started at.

XXXVI. To find the true bearing of a direction, when only local attraction partially prevents.

RULE.—Take the bearing and its reverse, and measure the distance; from each end of this distance take a bearing, its reverse, and distance, the ending of each of which must intersect, so that the three bearings and distances make a triangle; apply the differences of the bearings and their reverse as is hereafter mentioned, to get the true bearings.

XXXVII. To find the true bearing of a direction, when only local attraction prevents, by vibrating the needle of the instrument, or drawing it down fast.

RULE.—Get a right line between the objects; obtain a true bearing across this line; measure a line the nighest way and at right angles from this obtained bearing, to the right line between the objects; and measure a line on the obtained bearing, beginning at the beginning of the other measured line, and ending on the right line between the two objects. Then with the obtained bearing and the two taken distances, draw the following, viz. First, a meridian; second, the obtained bearing; third, the first measured line; fourth, the second measured line; fifth, a line of sufficient length to connect the endings of the drawn distances, which line, call the right line between the objects; from a scale of Chords, take 60 Degrees with the dividers, and with one foot at the ending of the last drawn distance, describe an arch from the right line between the two objects, across the angle that the first drawn distance crosses, to the obtained bearing; with the dividers measure this arch, lay it on the scale of Chords, and you have the difference between the obtained and required bearings; then apply this difference to the obtained bearing as follows, viz. If the required bearing lies to the right of the obtained one, and the obtained bearing is a N. E., or a S. W. one, Or if the required bearing lies to the left of the obtained one, and the obtained bearing is a N. W., or a S. E. one, the difference between the two bearings should be added to the obtained bearing to give the required one. And if the required bearing reads 90°, omit the letter that the nature of the case requires; but if it reads more than 90°, take 90° from it, and change the name of the letter that the nature of the case requires to an opposite name, and you have the bearing required. But if the required bearing lies to the left of the obtained one, and the obtained bearing is a N. E., or a S. W. one, or if the required bearing is a N. W., or a S. E. one, the difference between the two bearings should be subtracted from the obtained bearing to give the required one; and if the bearing thus found reads 90°, or if the two bearings do not happen to be in the same quarter of the compass, omit and change the letter that the nature of the case requires.

XXXVIII. To find the true bearings of a survey, when only local attraction partially prevents.

RULE.—Take the bearings, their reverse, and the distances, and in noting them let Att. stand for attraction, B. for bearing, R. B. for reverse bearing, D. for distance, Diff. for difference between the bearing and its reverse, and E. or W. at the right of the Diff. signify that the Att. is east, or west; and the Diff. *shows how much* the Att. is, and if the B. is greater than the *R. B. the Att. is of* the same name that the bearing's second let*er is of, but if the B. is* less than the R. B. the Att. is of an op-

posite name; and in noting the bearings, their reverse, and the distances, note the bearings first, and if S. &c. change it to an opposite name, note their reverse at the right of the bearings, and if S. &c. change them to an opposite name, to the right of the R. B. note the Diff., and to the right of the Diff. note the distances. Apply that Diff. that immediately follows that B. that has the least, or no Att., to the next following B. and as the rule for applying the Att. or Diff. dictates; and if the next Diff. is of the same name that the first applied Diff. is of, their sum should be applied to the next to be corrected B., but if it is of an opposite name their difference should be applied, and the same is to be observed in the application of the Att. of any two bearings; observing to call their difference of the same name that the greater Diff. is of, minding not to apply the last noted Diff. to the first noted B.

Rule for applying the Diff., or Att. If the to be corrected bearing is a N. W., or a S. E. one, and the Att. of the former B. is a W. Att. it should be added to the to be corrected B., but subtracted if it is a N. E., or a S. W. B; but if the Att. is an E. Att., it should be added to a to be corrected bearing, if it is a N. E., or a S. W. one, but subtracted, if it is a N. W., or a S. E. B.

☞ The example here given is one of actual experiment, taken by the Author in one of the Western States.

It is the Literati only that can form the estimate that will do justice to the preceding new and truly important rule.

EXAMPLE.

B. S.=N. 54°, 30′ W.=E.;—R. B. N.=N. 50°, 45′ W.=W.; Diff. and Att.=3°, 45′ W.; D. 4 ch. 07 links.
" N.= " 82°, 15′ " =W.;— " S.= " 83°, 30′ " =E.; " " " =1°, 15′ E.; D. 5 " 16 "
" " = " 80°, 30′ " = " ;— " " = " 80°, 30′ " = " ; " " " =0°, 00′ ; D. 8 " 38 "
" " = " 59°, 00′ " =E.;— " " = " 60°, 00′ " =W.; " " " =1°, 00′ W.; D. 8 " 01 "
" S.= " 42°, 00′ " = " ;— " N.= " 45°, 30′ " = " ; " " " =3°, 30′ E.; D. 4 " 91 "

NOTE. 1.—The preceding shows that the E. Att. is equal to the W. Att.

NOTE 2.—The following are the preceding example's bearings, corrected.

1. B. S. 54°, 30′ E.—D. 4 chains and 07 links.
2. B. N. 86°, 00′ W.—D. 5 " " 16 "
3. B. N. 83°, 00′ W.—D. 8 " " 38 "
4. B. N. 56°, 30′ E.—D. 8 " " 01 "
5. B. S. 45°, 30′ E.—D. 4 " " 91 "

NOTE 3.—The Att. of the first B. was first applied because the E. and W. Att. on the third B. was alike, hence why it reversed.

NOTE 4.—Now the Att. of the first B. is W. and the second B. is N. W., therefore the Att. is added to it; the second Diff. being E. and less than the first, therefore the Rem. should read W. and be added to the next B. because it is a N. W. B.; and because the next Att. or Diff. is W., the Diff. between the first and second Diff. and the third Diff. is added to the fifth B. it being a S. E. B.

NOTE 5.—In próof of the preceding Problem, I offer the following: From the bearings of a plot, let one draw a diagram, just as the bearings dictate, and if Att. intrudes on the bearings, the last drawn line deviates instead of just closing the diagram. But if you have the reverse of these bearings, get the corrected bearings, and draw a diagram thereby, and if the bearings their reverse, and the distances are correctly taken and proceeded with, the last drawn line just closes the diagram.

XXXIX. To survey a field from one station, at any place within the field, from which the several angles may be seen.

RULE.—Take the bearings of the Angles, and measure their distances from the Station; from these taken bearings and distances, draw a diagram, which divide into triangles, by drawing diagonals; the sum of the areas of these triangles is the area of the field; and to find the area of a triangle is shown by the following

NOTE.—From half the sum of the three sides subtract each side severally; multiply these three remainders and the said half sum continually together; then the square root of the last product will be the area of the triangle.

XL. Given, the minutes of a Field, to draw a Map of it, and to find the area by Construction.

RULE.—Draw a line to represent a Meridian, (i. e. a N. and S. line,) from which lay off a Bearing or Course of the first side of the field, from a line of Chords; and from a scale of equal parts *measure the length* of the side, and draw a line to represent it; *at the end of this line* draw a line parallel to the meridian line,

and then lay off the second side of the field as before taught; proceed in the same manner to draw parallel lines, and to lay off the several sides till the whole is protracted, divide this map into triangles by drawing diagonals, and the sum of the areas of these triangles is the area of the field.

NOTE 1.—In protracting a field, let the top of the paper be considered as North, the bottom South, the right hand East, and the left hand West; lay the course to the right or left of the meridian line, according as it is E. or W., and from the upper or lower part of the line, according as it is N. or S.

NOTE 2.—In all protractions, if the end of the last distance falls exactly on the point from which you began, the Course also being right, the field work and protraction are accurately performed; if not, an error must have been committed in one of them. In such cases, make a second protraction; if this agrees with the former, it is to be presumed the fault is in the field work; a resurvey is necessary.

XLI. To estimate the Distance of Objects on level ground, or at sea, having only the height given.

RULE 1.—To the earth's diameter (viz. 42056462 ft.) add the height of the eye, and multiply the sum by that height, then the square root of the product is the distance at which an object on the surface of the earth or water can be seen by an eye so elevated.

2. As objects are seen in a straight line, and that line is a tangent to the earth's surface, therefore, to find the distance of two elevated objects, when the right line joining them touches the earth's surface between those objects, (for instance, the line from the eye of the observer to the distance found by the first part of the rule, and from thence to the object,) work for each object separately, and the sum of the square roots of the products is the distance of the two objects from each other.

EXAMPLE.

How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated five feet above the surface?

$$\sqrt[2]{42056462+5\times5}= \text{- - - - - } 2{,}746 \text{ miles.}$$

$$\sqrt[2]{42056462+5280\times5280}=89{,}253 \text{ "}$$

Ans. 91,999 "

XLII. To estimate the Height of Objects on level ground, or at sea, having only the distance given.

RULE 1.—From the given distance, take the distance which the elevation of your eye above the surface will give, found by the last Problem.

2. Divide the square of the remainder in feet by 42056462 *ft.*, and the quotient will be the height required.

EXAMPLE.

Being on my return from a foreign voyage, and finding by my reckoning I was about 5½ leagues from Boston light house, it being in the dusk of evening, with my telescope I descried the lamp of the lighthouse in the horizon, at which time my eye was elevated 6 feet above the surface of the water. Now supposing my reckoning to be true, What is the height of the lighthouse above the water?

OPERATION.

5½ leagues=16,5 miles; then $16,5-\sqrt[2]{42056462+6\times6}=$ 13,943 miles; and 13,943 miles=73619 feet, nearly, and $\overline{73619\times73619}\div42056462=129$ feet, nearly. Ans.

MISCELLANY.

1. A man was hired 50 days on these conditions: that for every day he worked he was to have 75 cts, and for every day he was idle he was to forfeit 25 cts. How many days did he work, and how many was he idle, if he received $27,50 at the end of the time?—Had he worked every day, his wages would have been $37,50, i. e. $10 more than he received, but every day he was idle lessened his wages $,75+$,25=$1; consequently he was idle 10 days, hence 50—10=40 days at work.

2. A and B have the same income; A saves ⅛ of his, but B, by spending $30 per annum more than A, at the end of 8 years finds himself $40 in debt; What is their income, and what does each spend yearly?—Now $40÷8 yrs.=$5 yearly more than his income, and 5 of the $30 that B spends more than A, is the $5 yearly more than his income; hence $30—$5=$25=⅙ of either's income, therefore $200 their income; consequently, A spends $175, and B $205 per annum.

3. There is a fish whose head is four feet long; his tail is as long as his head and ½ the length of his body, and his body is as long as his head and tail. What is the length of the fish?—Its head=⅛ of it; the length of the body is ½ the length of the fish, hence 32 feet. Ans.

4. A man when he was married was 3 times as old as his wife; 15 years after, he was but twice as old as his wife. At what age was each married?—When they were married her age was 1 *year to his 3;* 15 years being added to their ages, hers is 2 years *to his 4; i. e. her age was* doubled, and his was ⅔ of what it *was, and 15 years doubled her* age, she was 15, and he was 45.

5. A lady has two silver cups, and only one cover. The first cup weighs 12 oz. If the first cup be covered, it will weigh twice as much as the second; but if the second cup be covered, it will weigh three times as much as the first. What is the weight of the cover, and of the second cup?—First cup oz. $\overline{12\times3}$ times=second cup and cover=36 oz.; then the 2 cups and cover=48 oz.; hence $\frac{2}{3}$ of 48 oz.=first cup and cover, and $\frac{1}{3}$ of 48 oz.=16 oz=second cup, then $48-\overline{16+12}=20$ oz.= the cover.

6. A general disposing his army into a square, found he had 231 over and above; but increasing each side with one soldier, he wanted 44 to fill up the square. Of how many men did his army consist?—$\overline{231+44+1}\div2=138$; then, $138\times138-44=$ 19000 men. Ans.

7. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form; and then the number in rank and file was double what it was at first; he was again reinforced with three times his second number of men, and after placing the whole in the same form as at first, his number in rank and file was 40 men each. How many men had he at first?

$\overline{\frac{3}{2}\times3+2}=\frac{8}{2},\times3+\frac{8}{2}=\frac{32}{2}$, then $\overline{40\times40\times2}\div32=100$. Ans.

8. A fellow said that when he counted his nuts two by two, three by three, four by four, five by five, and six by six, there was still an odd one; but when he counted them seven by seven they came out even. How many had he?

Thus, $2\times3\times4\times5\times6+1=721$. Ans.

9. A gentleman making his addresses in a lady's family, who had five daughters, was told that their father had made a will which imported that the first four of the girls' fortunes, were, together, to make £50000; the last four £66000; the three last, with the first, £60000; the three first, with the last, £56000; and the two first, with the two last, £64000, which, if he would unravel, and make it appear what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he should be welcome to her. What was Miss Harriet's fortune?

$$\left.\begin{array}{r} A+B+C+D=50000 \\ B+C+D+E=66000 \\ A+C+D+E=60000 \\ A+B+C+E=56000 \\ A+B+D+E=64000 \\ \hline \text{Sum, } 296000 \end{array}\right\}$$

Then, 296000÷4, the number of combinations,=74000, the sum of their fortunes;

Then, $A+B+C+D+E=74000$
And $A+B+D+E=64000$

Ans. Harriet's fortune=£10000

10. A said to B and C, give me $\frac{1}{3}$ of your money and I shall have £100; B said to A and C, give me $\frac{1}{3}$ of yours, and I shall have £100; and C said to A and B, give me $\frac{1}{4}$ of yours, and I shall have £100. How much money had each man?

11. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot; What would be the weight of such a tube, which would extend across the Atlantic, from Boston to London, estimating the distance at 1000 leagues?

First 1000 leagues=15840000 feet, and $15840000 \times \frac{1}{1625} = 9747\frac{69}{100}$ grains=1 lb., 8 oz., 6 pwt., $3\frac{69}{100}$ grains. Ans.

12. If the mean distances of the planets from the Sun in English miles are as follows: viz. Mercury, 36841468; Venus, 68891486; Earth, 95173000; Mars, 145014148; Jupiter, 494990976; Saturn, 907956130, and the Moon 240000 from the Earth, now as a cannon ball, at its first discharge, flies at the rate of a mile in 8 seconds, and sound at 1142 feet per second, at this rate, how long would a bullet be in passing from the earth to the Sun, and sound in moving from the Sun to Saturn?

95173000×8″=24 yrs., 52 ds., 7 h., 33 min., 20 sec. for the passage of the ball; and $\overline{907956130 \times 5280} \div 1142 = 133$ yrs., 41 d., 20 h. 55 m. $49\frac{42}{571}$ sec.=the sound's passage.

13. Light passes from the Sun to the Earth in about 8 minutes. How long would it be in passing from the Sun to Herschell's planet, supposing it to be 5000000000 miles? As 95173000 : 8 : : 5000000000 : 7h., 0m., 17s. 7 thirds.

14. If the diameter of the Sun is 890000 miles; Mercury's diameter 3000; Venus' 7924; the Earth's 7970; Mars' 7338; Jupiter's 94000; Saturn's 78000, and the Moon's 2182; What is the comparative *magnitude between* the Sun and the Earth, *and the Earth and all the others?*

SOLUTION OF PROBLEM 10.

A+B+C=A's.

$\overline{2+1+1=200\times3-1+3+1=300\times11-1+1+4=400\times3-1+3+1=300\times2=51)1500=£29\frac{7}{17}.=A's.}$

A+B+C=B's.

$\overline{1+3+1=300\times2-2+1+1=200\times7-1+1+4=400\times2-2+1+1=200=34)2200=£64\frac{12}{17}.=B's.}$

A+B+C=C's.

$\overline{1+1+4=400\times3-1+3+1=300\times5-2+1+1=200\times3-1+3+1=300\times2=51)3900=£76\frac{9}{17}.=C's.}$

The Sun$=\overline{89000}|^3 \div \overline{7970}|^3=1392499,52$ times larger than the Earth.

The Earth$=\overline{7970}|^3 \div \overline{3000}|^3=18,75$ times larger than Mercury.

The Earth$=\overline{7970}|^3 \div \overline{7924}|^3=1,0175$ times larger than Venus.

The Earth$=\overline{7970}|^3 \div \overline{5400}|^3=3,21$ times larger than Mars.

The Earth$=\overline{7970}|^3 \div \overline{2182}|^3=48,82$ times larger than the Moon.

Jupiter$=\overline{94000}|^3 \div \overline{7970}|^3=1640,62$ times larger than the Earth.

Saturn$=\overline{78000}|^3 \div \overline{7970}|^3=937,36$ times larger than the Earth.

15. If Paris, In France, be in 2° 20′ east long. from Greenwich, and Hallowell in 69° 42′ west long. from Greenwich, when it is noon at Paris, what time of day is it at Hallowell?—If 1° : 4 m. : : 69° 42′+2° 20′ : 4h. 48 sec. 8‴, then 12 hours less 4 h. 48 sec. 8‴=7 hours, 11 minutes, 52 seconds in the morning. Ans.

NOTE.—To reduce time to longitude: Multiply the hours by 15 for degrees; and divide the minutes and seconds respectively by 4 for m. and s.; and if two places be in the same longitude, their difference of latitude is the distance between them.

16. A man bought some lemons at 2 cents each, and $\frac{3}{4}$ as many at 3 cents each, and then sold them all at the rate of 5 cts. for 2, and thus gained 25 cents. How many lemons did he buy? He bought 4 at 2 cents each, as often as he bought 3 at 3 cents each, therefore he gave 17 cents for every 7 lemons$=2\frac{3}{7}$ cents each, but sold them at $2\frac{1}{2}$ cents each, then $2\frac{1}{2}-2\frac{3}{7}=\frac{1}{14}$ of a cent gained on 1 consequently $\frac{1}{14}\times 25=350$ lemons. Ans.

17. The stock of a cotton manufactory is divided into 32 shares, and owned equally by 8 persons, A, B, C, &c. A sells 3 of his shares to a ninth person, who thus becomes a member of the company, and B sells 2 of his shares to the company, who pay for them from the common stock. After this, what proportion of the whole stock does A own?—Each stockholder owns $\frac{4}{32}$ of the whole, and A reserved but $\frac{1}{32}$ of his, and 2 of B's shares being taken up by the company, hence only 30 shares in all, consequently A owns $\frac{1}{30}$ of the whole.

18. The river Po is 1000 feet broad, and 10 feet deep, and it runs at the rate of 4 miles an hour; in what time will it discharge a cubic mile of water, (reckoning 5000 feet to the mile) into the sea?—$1000\times 10\times 5000\times 4=200000000$ cubic feet it runs in an hour, and $\overline{5000}|^3=125000000000$ cubic feet in a mile; then, if 200000000 : 1 h. : : 125000000000 : 26 days, 1 hour. Ans.

19. The specific gravity of a cubic foot of quicksilver is 13600 *ounces* avoirdupois; and the specific gravity of a column of air

extending from the top of a barometer to the top of the atmosphere, (the column of air and barometer of an equal diameter,) is equal to a body of quicksilver extending from the top of the barometer in it and downward 30 inches; If the barometer stands at 30, what is the pressure of the atmosphere on a square foot? and what on the surface of the earth, if it has 62700000 square miles?—The height of the barometer is 2,5 feet; hence, 13600 oz.×2,5=2125 lbs. Ans. Then 4014489600 square feet=1 square mile, consequently 4014489600×62700000×2125=5348805580800000000000 lbs. Ans.

20. In measuring a height—at the first station, suppose the barometer to stand at 29, and the thermometer at 60; at the second station, the barometer at 28, and the thermometer at 40; What is the height of the second station, or the distance between the stations?

Rule, for the preceding, or any other of the same nature.—Multiply the mean height corresponding to the mean temperature of the two barometers (found in the Table) by the tenths of an inch in the difference of the two barometers, and this product by 30; divide this last product by the mean height of the two barometers, and the quotient will be the height required, with the error of a feet, only, if the height be less than a mile.

OPERATION.

Add $\left\{\begin{array}{l}\text{1st station}=29\\ \text{2d} \quad \text{"} \quad =28\end{array}\right\}\div 2=28,5=$barometers' mean height; Then, 29—28=1=10 tenths of an inch.

And add $\left\{\begin{array}{l}\text{1st station}=60\\ \text{2d} \quad \text{"} \quad =40\end{array}\right\}\div 2=50=$thermometers' mean height, against which, in the Table, you will find 90,66, the mean temperature of the two barometers. Then according to the Rule, 90,66×10×30÷28,5=954,3 feet, the second station is higher than the first. Ans.

21. A, B, C, and D, found a purse of shillings and each of them took a number at a venture, afterwards by comparing them together they found that if A took 25 from B, his number would be equal to what B had left, and if B took 30 from C, his number would be three times what C had left, and if C took 40 from D, his number would be double to what D had left, and if D took 50 from A, his number would be 3 times as much as A had left, and 5*s.* over. What number had each?—Suppose A had 94, then 94+25+25=144+30÷3,+30=88+40=128÷2,+40=104+50=154; 94—50=44,×3=132; 154—132=22,—5=*17=first error.* Suppose A had 106; +25+25=144,+30=*186÷2,+30=92+40=132÷2,+40=106+50=156; 106—50=56,×3=168; 168—156=12,+5=17=*second error.

then, $106\times17=1802$, and $94\times17=1598$; then $1802+1598=3400$, and $17+17=34$; then $3400\div34=100s.$=A's number. Hence $150s.$=B's number; $90s.$=C's; and $105s.$=D's. Ans.

22. A merchant tailor bought 40 yards of cloth $2\frac{1}{4}$ yards wide, but being wet it shrunk in length upon every 4 yards $\frac{1}{2}$ of $\frac{1}{4}$ of a yard, and in width 1 nail and $\frac{1}{2}$ on every yard and half; to line this cloth he bought baize 5 quarters wide, which, being made wet, did shrink the whole width on every 20 yards in length, and in width it shrunk half a nail. The number of yards of baize, used in lining this cloth, is required.—If yds. 4 : 2 n. : : 40 yds. : 1 yd. 1 qr., then yds. 40—1 yd. 1 qr.=38 yds. 3 qrs. in length after shrinking; also 2 yds. 0 qr. $1\frac{3}{4}$ n. in width after shrinking; then $38,75\times2,0109375=77,923828125$ square yds. remainder. Again yds. 20×5 qrs.=25 yds. before it shrinks, and $18,75\times1,21875=22,8515625$ yds. of baize after it shrinks. Then, if yds. 25 : 22,8515625 : : 77,923828125 : 71,227+ yds. of baize. Ans.

23. What will 19 tons, 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. cost at £19, $19s.$ $11d.$ $3qrs.$ per ton?

£19, $19s.$ $11\frac{3}{4}d.\times\overline{3\times6}=359l.$ $19s.$ $7\frac{1}{2}d.$=cost of 18 T.

18

	359	19	$7\frac{1}{2}$	
10 cwt.=$\frac{1}{2}$	19	19	$11\frac{3}{4}$	=price of one ton.
5 cwt.=$\frac{1}{2}$	9	19	$11\frac{7}{8}$	
4 cwt.=$\frac{4}{5}$	4	19	$11\frac{15}{16}$	
2 qrs.=$\frac{1}{8}$	3	19	$11\frac{19}{20}$	
1 qr.=$\frac{1}{2}$		9	$11\frac{159}{160}$	
14 lbs.=$\frac{1}{2}$		4	$11\frac{319}{320}$	
7 lbs.=$\frac{1}{2}$		2	$5\frac{639}{640}$	
$3\frac{1}{2}$ =$\frac{1}{2}$		1	$2\frac{1279}{1280}$	
2 =$\frac{4}{7}$			$7\frac{1279}{2560}$	
1 =$\frac{1}{2}$			$4\frac{1279}{4480}$	
			$2\frac{1279}{8960}$	

NOTE.—The sum of the several fractions is $8\frac{16641}{17920}d.$

£399, $19s.$ $5\frac{16641}{17920}d.$=Ans.

24. In a mixture of wine and cider, $\frac{1}{2}$ of the whole plus 25 gallons, is wine, and $\frac{1}{3}$ part less 5 gallons is cider. How many gallons of each kind in this mixture?—$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, therefore, 25 gals.—5 gals.=20 gals.=$\frac{1}{6}$, hence 120 gals. in all. Consequently, 85 gals. is wine, and 35 gals. is cider.

25. Suppose one farthing had been put out, at 6 per cent. per annum, compound interest, at the commencement of the Christian Era; What would it have amounted to in A. D. 1784; and suppose the amount to be in standard gold, allowing a cubic inch to be worth $53l.$ $2s.$ $8d.$, how large would the mass have been? *Any sum*, at 6 per cent. per annum, compound interest, doubles

itself in 11,89 years; then $1784 \div 11,89 = 150$ times the given sum is doubled. And

$\frac{\overline{2}|^{150-1}}{2-1} \times 1 =$ £1486716346568748209435714551509890767065-361, 11*s*. $3\frac{3}{4}$*d*.

Hence, 279808597221212304159795712329335942107 66 cubic inches, then if 355 : 113 : : $\overline{360 \times 69,5} = 7964$, earth's diameter; Then $360 \times 69,5 \times 7964 \times 1327,33 = 264482820122$ cubic miles in the globe $= 6727333730885474136883 2000$ cubic inches in the globe; and the cubic inches of gold, divided by the cubic inches in the globe, gives 415930899840288,8 times larger than the globe we inhabit, however incredible it may appear.

26. How many locks, whose wards differ, may be unlocked with a key of 6 several wards?

1st, $6 \div 1 = 6$ locks of 1 ward each.

$\frac{6 \times 5 = 30}{2 \times 1 = 2} = 15$ locks of 2 wards; $\frac{6 \times 5 \times 4 = 120}{1 \times 2 \times 3 = 6} = 20$, of 3 wards;

$\frac{3 \times 4 \times 5 \times 6 = 360}{4 \times 3 \times 2 \times 1 = 24} = 15$ locks, of 4 wards;

$\frac{2 \times 3 \times 4 \times 5 \times 6 = 720}{5 \times 4 \times 3 \times 2 \times 1 = 120} = 6$ locks of 5 wards;

$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720}{6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720} = 1$ lock of 6 wards;

2d, $5 \div 1 = 5$ locks of 1 ward each;

$\frac{4 \times 5 = 20}{2 \times 1 = 2} = 10$ locks of 2 wards; $\frac{3 \times 4 \times 5 = 60}{3 \times 2 \times 1 = 6} = 10$, of 3 wards.

And the same of the rest of the 2d class, also the same of the 3d, 4th, and 5th classes, the sum of which is 63 locks, Ans.

27. C owes D $1400, to be paid in 3 months; but D being in want of money, C pays him at the expiration of 2 months, $1000; how much longer than 3 months ought C, in equity, to defer the payment of the rest?—First, $\overline{1400 \times 3} - \overline{1000 \times 2} = 2200$, and $2200 \div \overline{1400 - 1000} = 5\frac{1}{2}$ months; then $5\frac{1}{2} - 3 = 2\frac{1}{2}$ months. Ans.

28. A retailer has 30 gallons of rum, whose specific gravity is 900; How much water must he add to reduce it to standard proof?

$925 \left\{ \begin{matrix} 1000 \\ 900 \end{matrix} \right] \begin{matrix} 25 \\ 75 \end{matrix} \Big\}$, As 75 : 25 : : 30 : 10 gals. of water. Ans.

29. What is the specific gravity of French brandy, consisting of 5 parts, measure, of rectified spirits of wine, and 3 parts water?—$\overline{850 \times 5} + \overline{1000 \times 3} = 7250$, and $7250 \div \overline{5 + 3} =$ *906,25=specific gravity.*

30. A cubic inch of glass weighs about 1,36 oz. Troy; ditto *of salt water ,5427 oz.*; ditto of brandy ,48927 oz. Suppose

then, a seaman has a gallon of brandy in a bottle, which weighs $4\frac{1}{2}$ lbs. Troy, out of water, and, to conceal it, throws it overboard into salt water. Will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water?

$4\frac{1}{2}$ lbs.=54 oz.=weight of bottle; then 54÷1,36=39,7059 cubic inches in the bottle, and 231+39,7059=270,7059 cubic inches in the bottle and brandy. Then 270,7059×,5427=146,912 oz.=weight of salt water occupied by the bottle and brandy. And ,48927×231=113,02 oz., and 113,02+54=167,02 oz.=weight of the bottle and brandy; then 167,02—146,912=20,11 oz. heavier that the same bulk of salt water, therefore the bottle and brandy will sink.

31. How much wine, at 4*s.* 6*d.* and at 5*s.* per gallon must be mixed with 6 gallons at 4*s.* and 6 gallons at 3*s.* per gallon, that the mixture may be worth 4*s.* 4*d.* per gallon?

Limited simples $\left\{\begin{array}{l}\text{6 gals. at } 4s.=24\\ \text{6 " " } 3s.=18\end{array}\right\}$

12 - - - - - - - - -)42=3*s.* 6*d.* rate of the limited simples.

Now the question may stand thus: How much wine at 4*s.* 6*d.* and 5*s.* per gallon, must be mixed with 12 gallons at 3*s.* 6*d.* per gallon, that the mixture may be worth 4*s.* 4*d.* per gallon?

$$4s.\ 4d.=52\left\{\begin{array}{l}42\\54\\60\end{array}\right]\begin{array}{l}2+8\\10\\10\end{array}\Bigg|\begin{array}{l}10\\10\\10\end{array}$$

As $10:\left\{\begin{array}{l}10\\10\end{array}\right\}::12:\left\{\begin{array}{l}\text{12 gals. at } 4s.\ 6d.\\ \text{12 " " } 5s.\end{array}\right\}$ per gal. Ans.

32. Two persons comparing their ages, find them, at present, in the ratio of 7 to 5, but, that 30 years ago they were as 2 to 1. What is the age of each?—Inversely, 7+5 : 30 : : 2+1 : 120=sum of their ages; Then, As 7+5 : 120 : : 7 : 70 yrs.=oldest, then 120—70=50 yrs.=youngest. Ans.

33. A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of marbles; B caught $\frac{3}{8}$ of them out of his hands, and C laid hold on $\frac{3}{10}$ more; D ran off with $\frac{6}{7}$ of what A had left, and the rest E afterwards secured slily for himself. Then A and C jointly fell upon B, who, in the conflict, let fall $\frac{1}{2}$ he had, which were equally picked up by D and E; B then kicked down C's hat, the contents of which A got $\frac{1}{4}$, B $\frac{1}{5}$, D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands; they with difficulty recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ apiece of the same. Upon this, they called a truce, and agreed that the $\frac{1}{5}$ left by A at first should be equally divided among them. How much of the prize, after this distribution, had each of the competitors?

1st. A got $\frac{2}{3}$, and B got $\frac{3}{8}$ of $\frac{2}{3}=\frac{1}{4}$; then C got $\frac{3}{10}$ of $\frac{2}{3}=\frac{1}{5}$.

Then $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$, and $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}$=what A had left. D then ran off with $\frac{6}{7}$ of $\frac{13}{60}=\frac{78}{420}$, and E secured $\frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}$; consequently A had none left. So at the end of the first scuffle, B had $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{78}{420}$, and E $\frac{13}{420}$ of them.

2d. B let fall $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$, and had $\frac{1}{8}$ left. D picked up $\frac{1}{2}$ of $\frac{1}{8}=\frac{1}{16}$, and E the same; then D had $\frac{78}{420}+\frac{1}{16}=\frac{139}{560}$, and E had $\frac{13}{420}+\frac{1}{16}=\frac{157}{1680}$. So at the end of the second scuffle, their shares stood thus: B $\frac{1}{8}$, C $\frac{1}{5}$, D $\frac{139}{560}$, and E $\frac{157}{1680}$.

3d. A got $\frac{1}{4}$ of $\frac{1}{5}=\frac{1}{20}$; B got $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$; and D got $\frac{2}{7}$ of $\frac{1}{5}=\frac{2}{35}$. Their sum is $\frac{1}{20}+\frac{1}{15}+\frac{2}{35}=\frac{79}{420}$, and $\frac{1}{5}-\frac{73}{420}=\frac{11}{420}$=what C had left. Then C took $\frac{1}{2}$ of $\frac{11}{420}=\frac{11}{840}$, and E the same. B then had $\frac{1}{8}+\frac{1}{15}=\frac{23}{120}$; D had $\frac{139}{560}+\frac{2}{35}=\frac{171}{560}$; and E had $\frac{157}{1680}+\frac{11}{840}=\frac{179}{1680}$. So at the end of the third scuffle, A had $\frac{1}{20}$, B $\frac{23}{120}$, C $\frac{11}{840}$, D $\frac{171}{560}$, E $\frac{179}{1680}$.

Lastly, $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$=what A and B last acquired. D struck out of their hands $\frac{3}{4}$ of $\frac{7}{60}=\frac{7}{80}$. Then A had $\frac{1}{4}$ of $\frac{1}{20}=\frac{1}{80}$, and B had $\frac{1}{4}$ of $\frac{1}{15}=\frac{1}{60}$ left. They recovered, in equal shares, $\frac{5}{8}$ of $\frac{7}{80}=\frac{7}{128}$; consequently A took $\frac{1}{2}$ of $\frac{7}{128}=\frac{7}{256}$, and B as much. Then A had $\frac{1}{80}+\frac{7}{256}=\frac{51}{1280}$, and B had $\frac{1}{60}+\frac{7}{256}+\frac{1}{8}=\frac{649}{3840}$. Then C, D, and E got each $\frac{1}{8}$ of $\frac{7}{80}=\frac{7}{640}$; consequently C had $\frac{11}{840}+\frac{7}{640}=\frac{323}{13440}$; D $\frac{171}{560}+\frac{7}{640}=\frac{1417}{4480}$; and E $\frac{179}{1680}+\frac{7}{640}=\frac{1579}{13440}$ of them when the truce was called; then each was to have $\frac{1}{5}$ of $\frac{1}{3}=\frac{1}{15}$ of the $\frac{1}{3}$ that was not scuffled after, consequently their shares stood as follows:

A's share was $\frac{51}{1280}+\frac{1}{15}=\frac{409}{3840}=\frac{2863}{26880}$;

B's " " $\frac{649}{3840}+\frac{1}{15}=\frac{905}{3840}=\frac{6335}{26880}$;

C's " " $\frac{323}{13440}+\frac{1}{15}=\frac{1219}{13440}=\frac{2438}{26880}$;

D's " " $\frac{1417}{4480}+\frac{1}{15}=\frac{5147}{13440}=\frac{10294}{26880}$;

E's " " $\frac{1579}{13440}+\frac{1}{15}=\frac{2475}{13440}=\frac{4950}{26880}$;

The numerators of the last fractions express their respective shares, which shares are the whole number of marbles and the common denominator to these fractions.

Ans. A had 2863; B, 6335; C, 2438; D, 10294; and E, 4950.

34. A tract of land, containing 100 acres, was purchased by A and B for $500, in the payment of which they paid equal sums of money. In the division of it, A, wishing to have his share off the side adjoining his own farm, agreed to receive so many acres less than one half as would make his part 75 cents more per acre than B's. How many acres had each, and what was the price per acre?

NOTE.—For the solution of questions of this nature, we have the following

RULE I.—Multiply the whole number of articles, or acres, by the difference in the two prices of one article, and subtract the *product* from the whole sum or price of the whole. Then *divide the remainder* by twice the difference in the price of one *article, calling* this the *first quotient*.

II. Multiply half the whole sum by the whole number of articles, and divide the product by the difference in the prices of one article, calling this the *second quotient*.

III. To the square of the *first quotient*, add the *second quotient*, and extract the square root of the sum, and from the square root, subtract the *first quotient*, and you will have the share of one, or the larger share. Subtract this share from the whole number of articles, and you will have the share of the other, or the smaller share.

OPERATION.

$\overline{500-100}\times,75\div,75\times2$ equal 283,33=*first quotient;* and $\overline{500\div2}\times100\div,75$ equal 33333,3333=*second quotient;* and $\sqrt[2]{283,33}^2+33333,3333=337,06$, which less 283,33 equal 53,73 acres, B's share; and 100—53,73=46,27 acres, A's share. And $250÷46,27=$5,40=A's per acre; and $250÷53,73=$4,65=B's per acre.

35. Two ships, A and B, sailed from a certain port at the same time. A sailed north 8 miles an hour, and B east 6 miles an hour. Required their distance asunder at every hour's end?

$\sqrt[2]{8\times8+6\times6}=10$ miles distant in 1 hour, and $10\times2=20$ miles in 2 hours, &c.

36. Suppose a ship sails from lat. 43° north, between north and east, till her departure from the meridian be 45 leagues, and the sum of her distance and difference of lat. to be 135 leagues, I demand her distance sailed, and latitude come to?

$\overline{135\times135-45\times45}\div\overline{135\times2}=60$ leagues, and $60\times3=180$ miles=3 degrees the difference of latitude, and 135—60=75 leagues the distance; and as the vessel is sailing from the equator, the latitude is increasing; therefore, 43°+3°=46 degrees north is the latitude come to.

37. Suppose a General would place 864 men in such a form that their length shall be to their breadth as 3 to 2, and the distance of each man, one from the other, 7 yards; how many men must there be in length, and how many in breadth, and how many square yards of ground do they stand on?

NOTE.—For the solution of questions of this nature we have the following

RULE I.—As the ratio in length is to the ratio in breadth, so is the *given number* to a fourth number, whose square root is the number in breadth; the same of the number in length.

II.—As 1 is to the distance, so is the number in breadth, less one, to a fourth number; the same of the number in length; the product of which is the square yards of ground they stand on.

OPERATION.

As 3 : 2 : : 864 : 576, and $\sqrt[2]{576}=24$ men=No. in breadth. Ans.
As 2 : 3 : : 864 : 1296, and $\sqrt[2]{1296}=36$ men=No. in length, Ans.
As 1 : 7 : : $\overline{24-1}$: 161 ; and, as 1 : 7 : : $\overline{36-1}$: 245 ;
And $245\times161=39445$ square yards they stand on. Ans.

38. Suppose a cask to measure diagonally (from the centre of the bung) 36 inches. What is its contents in gallons?

NOTE.—For the solution of questions of this nature, we have the following

RULE.—Divide the cube of the diagonal by 368, and the quotient is the answer.

$\overline{36\times36\times36}\div368=126,792+$ gallons. Ans.

39. There is a section of a tree 25 inches over; I demand the difference of the inscribed and circumscribed squares, and how far they differ from the area of the section?

$\overline{25\times25}-\overline{12,5\times12,5\times2}=312,5$ in. in the diff. of the squares. $\overline{25\times25}-\overline{25\times25\times11}\div14=134,125$ inches the circumscribed square is in area more than the section; and $\overline{25\times25\times11}\div14-\overline{12,5\times12,5\times2}=178,375$ in. the inscribed square, less than the area of the section.

40. There is a square pyramid, each side of whose base is 30 inches, and whose perpendicular height is 120 inches, to be divided by sections parallel to its base into 3 equal parts. Required the perpendicular height of each part?

$\overline{30\times30\times40}=36000$ the solidity in inches, and $\frac{2}{3}$ of it is 24000, $\frac{1}{3}$ is 12000. Then,

As $36000:\overline{120}|^3::\left\{\begin{matrix}24000\\12000\end{matrix}\right\}:\left\{\begin{matrix}1152000\\576000\end{matrix}\right.$ the cube root of each of which is $\left.\begin{matrix}104,8\\83,2\end{matrix}\right\}$ Then $120-104,8=15,2$, length of the thickest part, and $104,8-83,2=21,6$, length of the middle part, consequently 83,2 is the length of the top part.

41. Suppose a pole to stand on a horizontal plane 75 feet in height above the surface, At what height from the ground must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, resting on the upright part?

NOTE.—For the solution of questions of this nature, we have the following

RULE.—From the square of the length of the pole take the square of the base; divide the remainder by twice the length of the pole, and the quotient will be the perpendicular height at *which it must be* cut off?

$\overline{75\times75}-\overline{55\times55}\div\overline{75\times2}=17\frac{1}{3}$ feet from the ground. Ans.

42. For a job, a tinker appeared in a nettle,
So I asked him to make a flat bottomed kettle;
Let the top and the bottom diameters be
In just such proportion as five is to three;
Twelve inches in depth I proposed and no more,
And to hold in ale gallons seven less than a score.

Required the diameters.

$13\times282=3666$ cubic inches in 13 gallons, then $5\times3=15=$ product of the two numbers. $15\times3=45=$three times their product; $5-3=2=$diff. of the two numbers; $2\times2=4=$square of the diff., and $45+4=49=$the sum. Then $49\times,7854\times4=153,9384$, and $3666\div153,9384=23,8147$; then $\sqrt[2]{23,8147}=4,88$.

Then $\begin{cases} 4,88\times5=24,40+ \text{ inches,}=\text{larger diameter;} \\ 4,88\times3=14,64+ \text{ inches}=\text{smaller diameter.} \end{cases}$

43. What is the Hypothenuse and Perpendicular of a right angled triangle whose sides are in the ratio of 7 to 4, the base being 100 feet? $\sqrt[2]{7\times7-4\times4}=5,7445$ ratio of the base.

Then $\begin{cases} 5,7445:4::100:\ 69,68 \\ 5,7445:7::100:121,94 \end{cases}$ Ans.

44. A, B, and C join to buy a grindstone, 36 inches in diameter, which cost $\$3\frac{1}{5}$, and towards which A paid $\$1\frac{1}{6}$, and B $\frac{8}{20}$, and C the remainder. The waste hole for the spindle was five inches square. To what diameter ought the stone to be worn, when B and C severally begin to work with it, allowing for the hole, and A first grinding down his share, next B, and then C?

$\$1\frac{1}{6}=\frac{7}{20}$, then $\frac{7}{20}+\frac{8}{20}=\frac{5}{4}$, and $\frac{4}{4}-\frac{3}{4}=\frac{1}{4}=\frac{5}{20}=$C's share$=83\frac{1}{3}$ cents, and $\frac{8}{20}=\$1\frac{1}{3}=$B's; $\sqrt[2]{5\times5\times2}=7\frac{1}{14}$, the diameter of a circle that will circumscribe a 5 inch square. $7\frac{1}{14}\times7\frac{1}{14}\times11\div14=39,285$, the area of that circle whose diameter is $7\frac{1}{14}$ in.

Then $\sqrt[2]{1018,285-\overline{1018,285\times7\div20}+\overline{39,285\times7\div20}}=29,327+$ inches in diameter when B begins to work. Ans. 1.

And $\sqrt[2]{1018,285-\overline{1018,285\times3\div4}+\overline{39,285\times3\div4}}=19,013+$ inches, diameter when C begins to work. Ans. 2.

45. Three men are to carry a stick of timber 12 feet long, and of equal size from end to end; one man is to carry the hind end, and two are to carry the forward end with a lever. How far from the forward end must the lever be placed, that each may sustain an equal portion of the weight?—Two men sustain twice as much (if at the forward end) as the man at the hind end; therefore the two will be twice as near the centre; consequently three feet. Ans.

46. If a stick 16 feet long was to be carried by five men, four forward and one behind, the forward men must be one fourth as

far from the centre as the hind one, that is, they must be six feet from the forward end.

47. A tract of land is to be laid out in the form of an equal square, and to be enclosed with a post and rail fence, 5 rails high, so that each rod of fence shall contain 10 rails. How large must said square be to contain just as many acres as there are rails in the fence that encloses it?

Thus, 1 mile=320 rods; then $\overline{320\times320}\div160=640$ acres; then $320\times4\times10=12800$ rails. And, if 640 : 12800 : : 12800 : 256000 rails, which will enclose 256000 acres=20 miles square. Ans.

48. Required, the length of a thread, winding once round in every three feet of a cone 60 feet in height and 3 feet in diameter at the base.

Thus, $\overline{,7854\times3}|^2\times60\div3=141,372,\div60=2,3562$ feet, area of a mean diameter, and $\sqrt[2]{2,3562\div,7854}=1,732,\times3,14=5,43848$, circumference of the mean diameter, and the base to what each yard of the perpendicular height of the cone is the perpendicular of; and the hypothenuse of this base and perpendicular, when multiplied by the yards in 60 feet, is the answer required.

49. What fraction is that, to the numerator of which, if 1 be added the value will be $\frac{1}{3}$; but if 1 be added to the denominator, Its value will be $\frac{1}{4}$?

Suppose 5, then $\overline{5+1}\times3$ den.=18, +1 num.=19; and 5×4 den.=20; then 20—19=1, Error.

Suppose 8, then $\overline{8+1}\times3$ den.=27, +1 num.=28; and 8×4 den.=32; then 32—28=4, Error.

Then $\begin{cases}\overline{5\times4}-\overline{8\times1}\div\overline{4-1}=4, \text{ numerator required.}\\ \overline{18\times4}-\overline{27-1}\div\overline{4-1}=15, \text{ denominator required.}\end{cases}$

Ans. $\frac{4}{15}$.

NOTE.—The 25th Miscellaneous Question in Welch's Arithmetic, is one of an Algebraic nature; hence an arithmetical solution cannot be given.

SUNDRY TABLES.

I. *Specific Gravities of the Solar System.*

A Cubic Foot of	Absolute Weight.	Relative Weight.
The Sun,	11333 oz.	11,333
Mercury,	9166 "	9,166
Venus,	5733 "	5,733
Earth,	4500 "	4,500
Mars,	3286 "	3,286
Moon,	3092 "	3,092
Jupiter,	1042 "	1,042
Saturn,	406 "	0,406

II. *Shows the amount of $1, or £1, at interest.*

Rate per cent.	Half yearly payments.	Quarterly payments.
3	1.007445	1.011181
3.5	1.008675	1.013031
4	1.009902	1.014877
4.5	1.011126	1.016720
5	1.012348	1.018559
5.5	1.013567	1.020395
6	1.014781	1.022257
6.5	1.015993	1.024055
7	1.017204	1.025880

II. *Thermometer.*

Deg.	Feet.
32	86,86
35	87,49
40	88,54
45	89,60
50	90,66
55	91,72
60	92,77
65	93,82
70	94,88
75	95,93
80	96,99

IV. *Of Superficies.*

A figure having equal sides and angles is a	
3	Trigon; ,433013
4	Tetragon; 1,
5	Pentagon; 1,720477
6	Hexagon; 2,589076
7	Heptagon; 3,633959
8	Octagon; 4,828427
9	Enneagon; 6,181827
10	Decagon; 7,694209
11	Endecagon; 8,51425
12	Dodecagon; 9,330125

NOTE.—If the square of a side, of either figure, in Table IV, be multiplied by the number at the right of it, the product is its true area.

V. *Of Solids and their Superficies.*

Names of the Bodies.	Solidity.	Superficies.
Tetraedron,	0.11785	1.73205
Hexaedron,	1.	6.
Octaedron,	0.4714	3.464
Eicosiedron,	2.181695	8.66025
Dodacaedron.	7.663119	20.6457

NOTE.—All like solid bodies being in proportion to one another as the cubes of their like sides, the solid content of any of the bodies named in Table V, may be found by multiplying the cubes of their sides by the numbers in the second column, under *Solidity*; and their superficies by multiplying the squares of their sides into the numbers in the third column, under *Superficies*.

VII. *A Table of the Proportions which the following Solids have to the Cube and Cylinder, having the same Base and Altitude.*

	Solid Inches.
1. A *Cube*, whose side is 12 inches, contains	1728
2. A *Prism*, having an equilateral triangle, whose side is 12 inches for its Base, and its Altitude 12 inches, contains - - - - - -	784,24
3. A *Square Pyramid*, whose height and the side of its base are each 12 inches, is $\frac{1}{3}$ of the above cube, and therefore contains - - -	576
4. A *Triangular Pyramid*, whose height and side of its triangular base are each 12 inches, is near $\frac{1}{7}$ of the cube, and contains - - -	249,413
5. A *Cylinder*, whose diameter and height are each 12 inches, is $\frac{11}{14}$ of the above cube, and contains	1357,17
6. A *Sphere* or *Globe*, whose axis or diameter is 12 inches, equal to the side of the cube, is $\frac{11}{21}$ of it, and contains. - - - - - - -	904,78
7. A *Cone*, whose base and altitude are each 12 inches, equal to the side of the cube, is $\frac{5}{19}$ of it, and contains - - - - - - -	452,38829
8. A *Parabolic Conoid*, whose diameter at the base, and height are each 12 inches, being $\frac{1}{2}$ its circumscribing cylinder, contains - - -	678,583
9. A *Hyperbolic Conoid*, whose height and diameter at the base, are each 12 inches, is $\frac{5}{12}$ of its circumscribing cylinder, and contains - -	565,49
10. A *Parabolic Spindle*, whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing cylinder, and contains - -	723,824

Hence arises a different method of finding their contents.

General Rule.—If the base of the solid whose content you would find, be rectilinear, consider it as a *Parallelopipedon;* if curved, as a *Cylinder;* and find the content accordingly; Then take such a part of the content, thus found, as is specified in the preceding Table, which, if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubic feet.

VI. *Of Circles.*

If the diameter of any circle

be { multiplied / divided } by { 3,14159 / ,31831 } { the product / the quotient } is the circumference; Or if it

be { multiplied / divided } by { ,886227 / 1,128379 } { the product / the quotient } is the side of an equal square; Or if it

be { multiplied / divided } by { ,866024 / ,1547 } { the product / the quotient } is the side of an equilateral triangle inscribed; Or if it

be { multiplied / divided } by { ,707016 / 1,414213 } { the product / the quotient } is the side of the square inscribed.

VII. *Shows the Specific Gravities of several solid and fluid Bodies, in Avoirdupois weight.*

A CUBIC FOOT OF	OUNCES.	A CUBIC FOOT OF	OUNCES.
Platina, rendered malleable and hammered,	20170	Brick,	2000
Very fine Gold,	19637	Live Sulphur,	2000
Standard Gold,	18888	Nitre,	1900
Moidore Gold,	17140	Alabaster,	1875
Guinea Gold,	17793	Dry Ivory,	1825
Quicksilver,	13600	Brimstone,	1800
Lead,	11325	Solid subs. Gunpowder,	1745
Fine Silver,	11087	Alum,	1714
Standard Silver,	10535	Ebony,	1117
Rose Copper,	9000	Human Blood,	1054
Copper,	8843	Amber,	1030
Plate Brass,	8000	Cows' Milk,	1030
Steel,	7852	Sea Water,	1030
Cast Brass,	7850	Pure Water,	1000
Iron,	7645	Red Wine,	993
Block Tin,	7321	Oil of Amber,	978
Cast Iron,	7135	Proof Spirits,	925
Lead Ore,	6800	Dry Oak,	925
Copper Ore,	3775	Olive Oil,	913
Diamond,	3400	Loose Gunpowder	872
Chrystal Glass,	3150	Spirits of Turpentine,	864
White Marble,	2707	Alcohol or Pure Spirit,	850
Black "	2704	Elm and Ash,	800
Rock Chrystal,	2658	Oil of Turpentine,	772
Green Glass,	2620	Dry Crab Tree,	765
Clear Glass,	2600	Ether,	732
Stone { Flint,	2582	White Pine,	569
Stone { Paving,	2570	Sassafras wood,	482
Stone { Cornelian,	2568	Cork,	240
Stone { Free,	2352	Common Air,	$1\frac{25}{100}$
		Inflammable Air,	$0\frac{12}{100}$

IX. *Shows the amount of 1£, or $1, from 1 year to 50 years.*

Y'rs.	3 per cent.	3 1-2 per cent.	4 per cent.	4 1-2 per cent.	5 per cent.	5 1-2 per cent.	6 per cent
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000	1,0550000	1,0600000
2	1,0609000	1,0712250	1,0816000	1,0920250	1,1025000	1,1130250	1,1236000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576250	1,1742413	1,1910160
4	1,1255088	1,1475230	1,1698585	1,1925186	1,2155062	1,2388245	1,2624796
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762815	1,3069598	1,3382256
6	1,1940523	1,2292553	1,2653190	1,3022601	1,3400956	1,3788426	1,4185191
7	1,2298738	1,2722792	1,3159317	1,3608618	1,4071004	1,4546789	1,5036302
8	1,2667700	1,3168090	1,3685690	1,4221006	1,4774554	1,5346862	1,5938480
9	1,3047731	1,3628973	1,4233118	1,4860951	1,5513282	1,6190939	1,6894789
10	1,3439163	1,4105987	1,4802842	1,5529694	1,6288946	1,7081440	1,7908476
11	1,3842338	1,4599697	1,5394546	1,6228530	1,7103393	1,8020919	1,8982985
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563	1,9012069	2,0121964
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8856491	2,0057732	2,1329282
14	1,5125897	1,6186945	1,7316764	1,8519449	1,9799316	2,1160907	2,2609039
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789281	2,2324756	2,3965581
16	1,6047064	1,7339860	1,8729812	2,0223701	2,1828745	2,3552617	2,5472716
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183	2,4848011	2,6927727
18	1,7024330	1,8574892	2,0258161	2,2084787	2,4066192	2,6214652	2,8543391
19	1,7535060	1,9225013	2,1068491	2,3078603	2,5269502	2,7656458	3,0255995
20	1,8061112	1,9897888	2,1911231	2,4117140	2,6532977	2,9177563	3,2071355
21	1,8602945	2,0594314	2,2787680	2,5202411	2,7859625	3,0782329	3,3995636
22	1,9161034	2,1315115	2,3699187	2,6336520	2,9252607	3,2475357	3,6035374
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715237	3,4261502	3,8197496
24	2,0327941	2,2833284	2,5633041	2,8760138	3,2250999	3,6145885	4,0489346
25	2,0937779	2,3632449	2,6658363	3,0054344	3,3863549	3,8133910	4,2918707
26	2,1565912	2,4459585	2,7724697	3,1406790	3,5556726	4,0231279	4,5493829
27	2,2212890	2,5315671	2,8833685	3,2820095	3,7334563	4,2443999	4,8223459
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291	4,4778419	5,1116867
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356	4,7241232	5,4183879
30	2,4272624	2,8067937	3,2433975	3,7453181	4,3219423	4,9839499	5,7434912
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380394	5,2580671	6,0881007
32	2,5750827	3,0067075	3,5080587	4,0899810	4,7649414	5,5472608	6,4533867
33	2,6523352	3,1119423	3,6483811	4,2740301	5,0031885	5,8523600	6,8405899
34	2,7319053	3,2208603	3,7943163	4,4663615	5,2533479	6,1742398	7,2510253
35	2,8138624	3,3335904	3,9460889	4,6673478	5,5160152	6,5138230	7,6860868
36	2,8982783	3,4502661	4,1039325	4,8773784	5,7918101	6,8720832	8,1472520
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069	7,2500478	8,6360871
38	3,0747834	3,6960113	4,4388134	5,3262192	6,3854772	7,6488004	9,1542523
39	3,1670269	3,8253717	4,6163659	5,5658990	6,7047511	8,0694844	9,7035074
40	3,2620377	3,9592597	4,8010206	5,8164645	7,0399887	8,5133060	10,2857178
41	3,3598988	4,0978337	4,9930614	6,0782054	7,3919881	8,9815378	10,9028608
42	3,4606958	4,2412579	5,1927838	6,3517246	7,7615875	9,4755224	11,5570325
43	3,5645167	4,3897020	5,4004952	6,6375522	8,1496669	9,9969761	12,2504547
44	3,6714522	4,5433415	5,6165150	6,9362421	8,5571502	10,5464933	12,9854817
45	3,7815957	4,7023585	5,8411756	7,2483730	8,9850077	11,1265504	13,7626109
46	3,8950436	4,8669411	6,0748236	7,5745497	9,4342581	11,7385217	14,5883673
47	4,0118949	5,0372840	6,3168166	7,9154045	9,9059710	12,3841404	15,4636693
48	4,1322518	5,2135889	6,5694892	8,2715977	10,4012696	13,0652681	16,3914894
49	4,2562193	5,3960645	6,8322688	8,6438196	10,9213331	13,7838579	17,3749788
50	4,3830059	5,5849268	7,1055596	9,0327915	11,4673697	14,5410000	18,2174775

Shows the present value of £1, or $1, at the end of any number of years, from 1 to 40.

Years.	4 per cent.	4 1-2 per cent.	5 per cent.	5 1-2 per cent.	6 per cent.
1	,961538	,956938	,952381	,947867	,943396
2	,924556	,915730	,907030	,898513	,889996
3	,888996	,876297	,863838	,851728	,839619
4	,854804	,838561	,822702	,807397	,792093
5	,821927	,802451	,783526	,765392	,747258
6	,790314	,767896	,746215	,725587	,704960
7	,759918	,734828	,710681	,687869	,665057
8	,730690	,703185	,676839	,652125	,627412
9	,702587	,672904	,644609	,618253	,591898
10	,675564	,643928	,613913	,586153	,558394
11	,649581	,616199	,584679	,573733	,562787
12	,624597	,589664	,556837	,526903	,496969
13	,600574	,564271	,530321	,499580	,468839
14	,577475	,539973	,505068	,473684	,442301
15	,555264	,516720	,481017	,449141	,417265
16	,533908	,494409	,458311	,425979	,393647
17	,513373	,473176	,436297	,403830	,371364
18	,493628	,452800	,415521	,382932	,350343
19	,474642	,433302	,395734	,363123	,330513
20	,456387	,414643	,376889	,344346	,311804
21	,438833	,396787	,358942	,326568	,294155
22	,421955	,379701	,341850	,309677	,277505
23	,405726	,363350	,325571	,293684	,261797
24	,390121	,347703	,310068	,278523	,246978
25	,375117	,332731	,305303	,269150	,232998
26	,360689	,318402	,281241	,250525	,219810
27	,340816	,304691	,267848	,237608	,207368
28	,333477	,291571	,255094	,225362	,195630
29	,320651	,279015	,242946	,213715	,184556
30	,308309	,267000	,231377	,202743	,174110
31	,290460	,255502	,220359	,192307	,164255
32	,285058	,244500	,209866	,182411	,154957
33	,274094	,233971	,199872	,173029	,146186
34	,263552	,223896	,190355	,164133	,137912
35	,254415	,214251	,181290	,155692	,130105
36	,243669	,205028	,172057	,147399	,122741
37	,234297	,196299	,164436	,140114	,115793
38	,225285	,187750	,156605	,132893	,109182
39	,216671	,179665	,149148	,126075	,103002
40	,208289	,171929	,142046	,110608	,097170

XI. *Shows the amount of £1, or $1 annuity, from 1 year to 40 years.*

Years.	4 per cent.	4 1-2 per cent.	5 per cent.	5 1-2 per cent.	6 per cent.
1	1,	1,	1,	1,	1,
2	2,04	2,045	2,05	2,055	2,06
3	3,1216	3,137025	3,1525	3,16802	3,1836
4	4,24646	4,278191	4,310125	4,34226	4,374616
5	5,416322	5,470710	5,725631	5,58109	5,637093
6	6,632975	6,716892	6,801913	6,888051	6,975318
7	7,898294	8,019152	8,142008	8,266894	8,393837
8	9,214266	9,380014	9,549109	9,721573	9,897467
9	10,582795	10,802114	11,026564	11,256259	11,491315
10	12,006107	12,288200	12,577892	12,875354	13,180794
11	13,486351	13,841179	14,206787	14,583498	14,971642
12	15,025805	15,464032	15,917126	16,385590	16,869940
13	16,626838	17,159913	17,712983	18,286798	18,882132
14	18,291911	18,932109	19,598632	20,292572	21,015064
15	20,023588	20,784054	21,578563	22,408663	23,275968
16	21,824531	22,719337	23,657492	24,641140	25,672527
17	23,697512	24,741707	25,840366	26,996402	28,212879
18	25,645413	26,855084	28,132385	29,481205	30,905652
19	27,671229	29,063562	30,520004	32,102671	33,759991
20	29,778078	31,371423	33,065954	34,868318	36,785590
21	31,969202	33,783137	35,719252	37,786075	39,992725
22	34,247979	36,303378	38,505214	40,864309	43,392289
23	36,617888	38,937030	41,430475	44,111846	46,[illegible]
24	39,088604	41,689196	44,501999	47,537998	[illegible]
25	41,645908	44,565210	47,727099	51,152[illegible]	[illegible]510
26	44,311745	47,570645	51,113454	54,96[illegible]	59,156381
27	47,084214	50,711334	54,669126	58,9[illegible]109	63,705763
28	49,967583	53,993333	58,402583	63,233510	68,528109
29	52,966286	57,423033	62,322[illegible]	67,711353	73,639796
30	56,084938	61,007067	66,438[illegible]47	72,435478	79,058183
31	59,328335	64,752388	70,[illegible]0790	77,419429	84,801674
32	62,701469	68,666245	75,298829	82,677498	90,889775
33	66,209527	72,756226	80,063771	88,224760	97,343161
34	69,857908	77,030[illegible]	85,066959	94,077122	104,183751
35	73,652225	81,49661[illegible]	90,320307	100,251363	111,434776
36	77,598314	86,103[illegible]	95,836323	106,765188	119,120863
37	81,702246	91,04[illegible]344	101,628139	113,637274	127,268114
38	85,970336	96,[illegible]205	107,709546	120,887324	135,904201
39	90,409150	10[illegible]64424	114,095025	128,536127	145,058453
40	95,025516	10[illegible]020323	120,799774	136,605614	154,761961

II. *Shows the present worth of £1, or $1 annuity, from 1 year to 40 years.*

Years.	4 per cent.	4 1-2 per cent.	5 per cent.	5 1-2per cent.	6 per cent.
1	0,96154	0,95694	0,95238	0,94786	0,94339
2	1,88609	1,87267	1,85941	1,84630	1,83339
3	2,77509	2,74896	2,72325	2,69790	2,67301
4	3,62989	3,58752	3,54595	3,49862	3,46510
5	4,45182	4,38997	4,32948	4,25759	4,21236
6	5,24814	5,15787	5,07569	4,97699	4,91732
7	6,00205	5,89270	5,78637	5,65888	5,58238
8	6,73274	6,59689	6,46321	6,30522	6,20979
9	7,43533	7,26879	7,10782	6,91786	6,80169
10	8,11089	7,91272	7,72173	7,49856	7,36008
11	8,76048	8,52892	8,30641	8,04898	7,88687
12	9,38507	9,11858	8,86325	8,57070	8,38384
13	9,98565	9,68285	9,39357	9,06522	8,85268
14	10,56312	10,22282	9,89864	9,53395	9,29498
15	11,11839	10,73954	10,37966	9,97824	9,71225
16	11,65229	11,23401	10,83777	10,39936	10,10589
17	12,16567	11,70719	11,27407	10,79852	10,47726
18	12,65929	12,15099	11,68958	11,17687	10,82760
19	13,13394	12,59329	12,08532	11,53549	11,15811
20	13,59032	13,00793	12,46221	11,87541	11,46992
21	14,02916	13,40472	12,82115	12,19760	11,76407
22	14,45111	13,79442	13,16300	12,50299	12,04158
23	14,85684	14,14777	13,48807	12,79245	12,30338
24	15,24696	14,49548	13,79864	13,06682	12,55035
25	15,62208	14,82821	14,09394	13,36880	12,78335
26	15,98277	15,14661	14,37518	13,57338	13,00316
27	16,32959	15,45130	14,64303	13,80702	13,21053
28	16,66306	15,74287	14,89813	14,02848	13,40616
29	16,98371	16,02189	15,14107	14,23838	13,59072
30	17,20202	16,28889	15,37245	14,43733	13,76483
31	17,58849	16,54439	15,59281	14,62590	13,92908
32	17,87355	16,78889	15,80268	14,80463	14,08398
33	18,14766	17,02286	16,00255	14,97404	14,22917
34	18,41120	17,24676	16,19290	15,13461	14,36613
35	18,66461	17,46101	16,37419	15,28680	14,49533
36	18,90828	17,66604	16,54685	15,43105	14,61722
37	19,14258	17,86224	16,71129	15,56779	14,73211
38	19,36787	18,04999	16,86789	15,69740	14,84048
39	19,58448	18,22965	17,01704	15,82024	14,94270
40	19,79277	18,40158	17,15509	15,93667	15,03913

XIII. *Annuities that £1, or $1 will purchase, from 1 year to 40 years.*

Years.	4 per cent.	4 1-2 per cent.	5 per cent.	5 1-2 per cent.	6 per cent.
1	1,04	1,045	1,05	1,055	1,06
2	,5302	,534	,5378	,54162	,54544
3	,36035	,39377	,36721	,37065	,37411
4	,27549	,27874	,28201	,28582	,28859
5	,22463	,22779	,23097	,23487	,23739
6	,19076	,19388	,19702	,20092	,20336
7	,16661	,16970	,17282	,17671	,17913
8	,14853	,15161	,15473	,15859	,16103
9	,13449	,13757	,14069	,14455	,14702
10	,12329	,12638	,12950	,13334	,13587
11	,11415	,11725	,12039	,12424	,12679
12	,10655	,10967	,11282	,11667	,11927
13	,10014	,10327	,10645	,11031	,11296
14	,09467	,09782	,10102	,10489	,10758
15	,08994	,09311	,09624	,10022	,10296
16	,08582	,08901	,09227	,09620	,09895
17	,08220	,08542	,08870	,09260	,09544
18	,07899	,08224	,08555	,08947	,09235
19	,07614	,07941	,08274	,08699	,08962
20	,07359	,07688	,08024	,08427	,08718
21	,07128	,07460	,07800	,08198	,08500
22	,06920	,07254	,07597	,07998	[illegible]
23	,06731	,07068	,07414	,07825	[illegible]
24	,06559	,06899	,07247	,07653	[illegible]
25	,06401	,06744	,07095	[illegible]	,07823
26	,06257	,06602	[illegible]	,07367	,07690
27	,06124	,06472	[illegible]	,07242	,07570
28	,06001	,06352	[illegible]	,07128	,07459
29	,05888	,06241	[illegible]	,07023	,07358
30	,05783	,06139	[illegible]	,06926	,07272
31	,05685	,06044	[illegible]	,06837	,07179
32	,05595	[illegible]	[illegible]	,06754	,07100
33	,05510	[illegible]	,06249	,06678	,07027
34	,05431	[illegible]	,06175	,06607	,06959
35	,05358	[illegible]	,06107	,06541	,06899
36	,05289	[illegible]	,06043	,06480	,06839
37	,05224	[illegible]	,05984	,06423	,06785
38	,05163	[illegible]	,05928	,06370	,06735
39	,05106	[illegible]	,05876	,06321	,06689
40	,05052	[illegible]	,05828	,06274	,06646

ARITHMETIC ODDITIES.

1. $99\frac{9}{9}=4$ nines, counting 100; and $11\frac{1}{1}=4$ ones, counting 12.

2. $\overline{35+29}=64$, $+8+7+1=$the 9 figures so placed as to count 80.

And $\left\{\begin{matrix}4\ 3\ 8\\9\ 5\ 1\\2\ 7\ 6\end{matrix}\right\}=$the 9 figures so placed as to count 15, several ways.

3. If a body of 45 lbs. weight be divided into 4 such parts, that one of which will weigh 1 lb., another 3 lbs., another 19 lbs., and the other 27 lbs., they may be so used as weights in a balance, that any number of pounds, from 1 lb. to 40 lbs. may be weighed by them.

And the number 45 may be divided into 4 such parts, that, if to the first part 2 be added, from the second part 2 be subtracted, the third multiplied by 2, and the fourth divided by 2, the several results will be equal. Ans. 8=1st, 12=2d, 5=3d, and 20=4th.

4. A bought 20 sheep for 20*s*., giving 4 pence for lambs, 1*s*. for dams, and 30 pence for wethers. How many of each kind did he buy?—Ans. 4 wethers, 7 dams, and 9 lambs.

5. B bought 100 animals for $100, giving $10 for oxen, $3 for hogs, and 50 cents for sheep. How many of each kind did he buy.—Ans. 5 oxen, 1 hog, and 94 sheep.

6. A sea captain, on a voyage with a cargo of 15 horses and 15 mules, being overtaken by a storm, found it necessary to lighten the ship, in order to save her, by throwing half of the cargo overboard; now the captain, being a shrewd man, and anxious to save the horses, because they were his, proposed to his mate, who owned the mules, to place the whole in a row and to throw every ninth beast overboard. How should they be placed so as to save the horses?—Ans. 1st 4 h., 2d 5 m., 3d 2 h., 4th 1 m., 5th 3 h., 6th 1 m., 7th 1 h., 8th 2 m., 9th 2 h., 10th 3 m., 11th 1 h., 12th 2 m., 13th 2 h., 14th 1 m.

7. If two of the nine figures be so placed as to count units and tens, and their sum subtracted from them, one figure of the result being known and subtracted from 9, discovers the other figure of the result.

8. To tell any number thought of.—Desire a person to think of a number and multiply it by 3; then if the product be an even number, divide it by 2; but if it be an odd number, add 1, and then divide it by 2. Let him multiply this quotient by 3, and if the product be even divide it by 2; if not, add 1, and then divide by 2, as before. Desire him to divide the last quotient by

9, and mention to you the quotient thence arising, regardless of the remainder.

You must multiply this quotient by 4, and if you added 1 before dividing the *first time* by 2, add 1 to the product; and if you added 1 before dividing the *second time* by 2, add 2 to this product. If the quotient cannot be divided by 9, you will add the numbers you added before dividing, calling the first 1, and the second 2, which will produce the number thought of.

A CONDENSED VIEW
OF THE PRECEDING.

After a familiarity with the pages of this work one may *yet* discover in it entirely new, very interesting and highly beneficial principles.

Its First Division teaches in six several instances the manner of reducing the art of multiplying to less than half the trouble that attends the *general* method; in it, the tables of weights and measures, are laid down on a new and improved plan, &c. &c.

The Second and Third Divisions illustrate many distinct principles, of much interest and too numerous to be *here* treated of.

The Fourth and Fifth Divisions, in almost every solution of their questions, illustrate distinct principles of much importance; and the reason of the solutions, also the classification of the questions and their different principles may be readily perceived by a little consideration.

The Sixth Division contains principles of the utmost importance and are such as *cannot* be found in common works on Arithmetic; and its thirty-eighth section is of such worth that a due estimation of it cannot be made; without it, a Surveyor of land cannot be master of his art; and with it, he can redress the wrongs done to either party by relying on local attracted bearings, and ever proceed with exact accuracy in surveying, to the great advantage of his employers, and honor to himself. It may be considered the discovery of importance that men of science have thought impossible to discover.

The Seventh Division is made up of questions and their solutions; and the eighth Division contains sundry tables of not a common nature; and the ninth is that of Arithmetic Oddities.

All of the preceding named matter is a select collection *only*, as deduced, arranged, and given by a person of twelve years study in Mathematics, an experienced teacher, and been nearly three years in its compilation.

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ARITHMETI

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JOSEPH WARREN, Esqr. of *Bur*
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